
Lecture 10: Bottom-up Analysis

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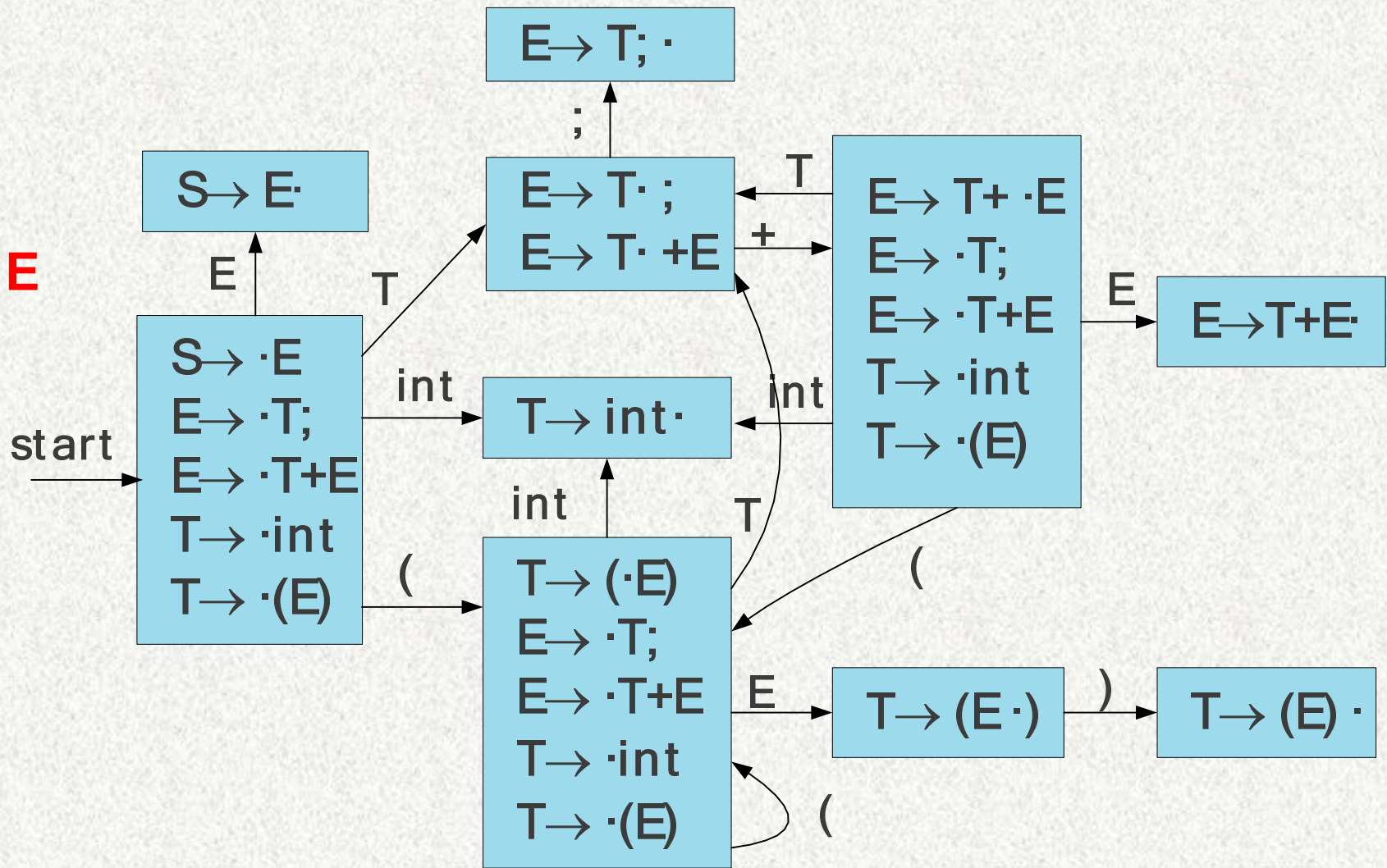
School of Computer Science E301



LR(0) parser

LR(0) Parsing

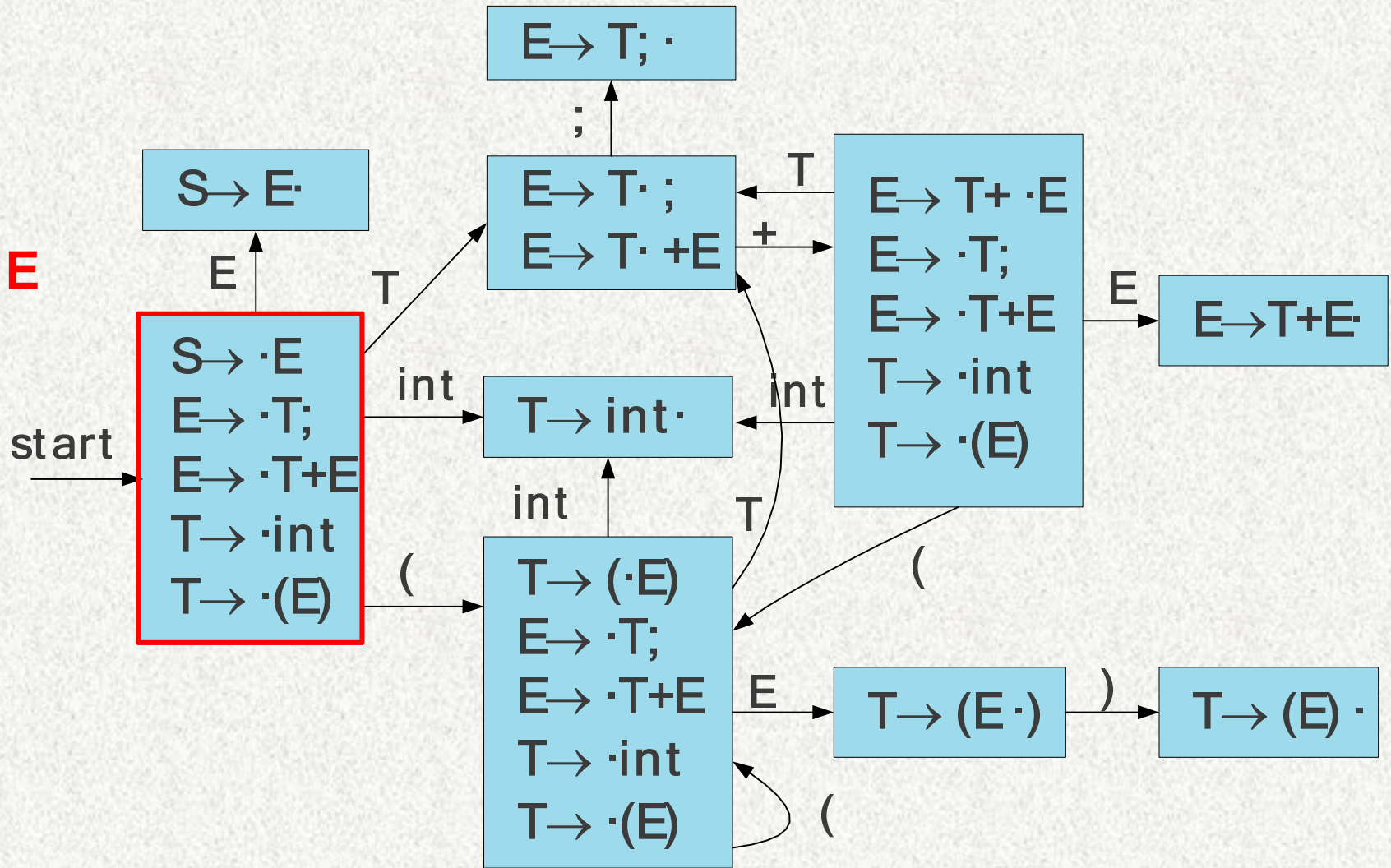
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



int	+	(int	+	int	;)	;
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LR(0) Parsing

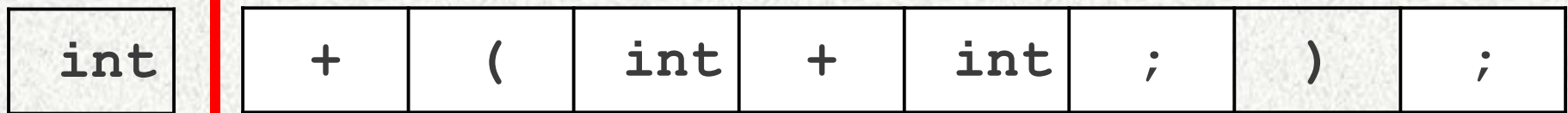
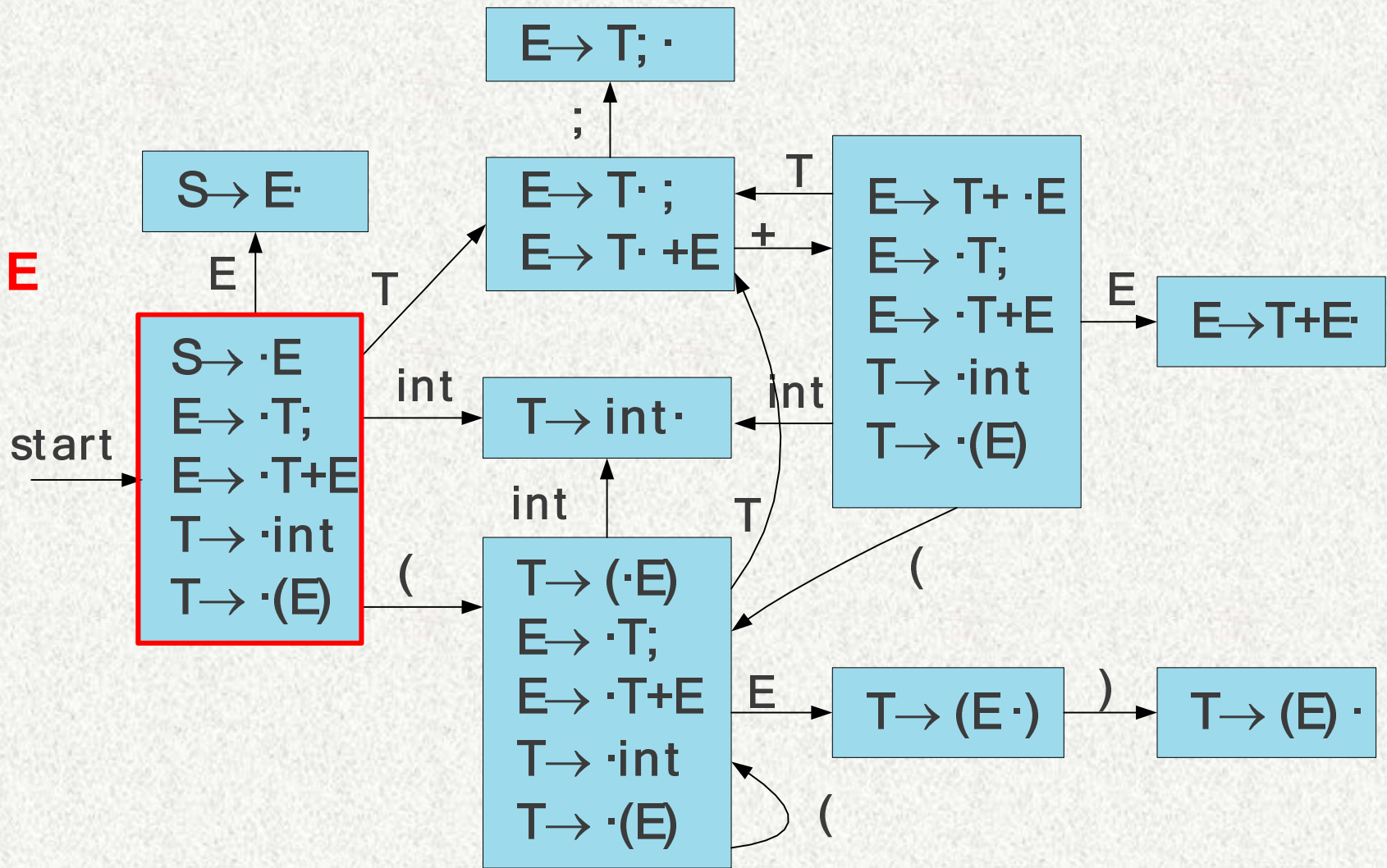
$S \rightarrow E$
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 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



int	+	(int	+	int	;)	;
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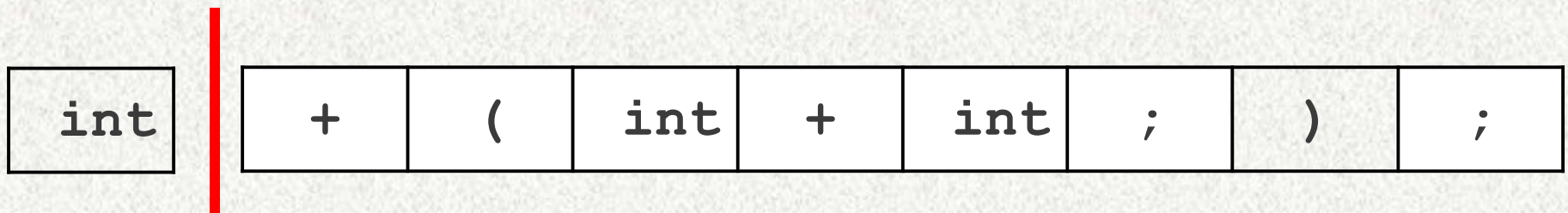
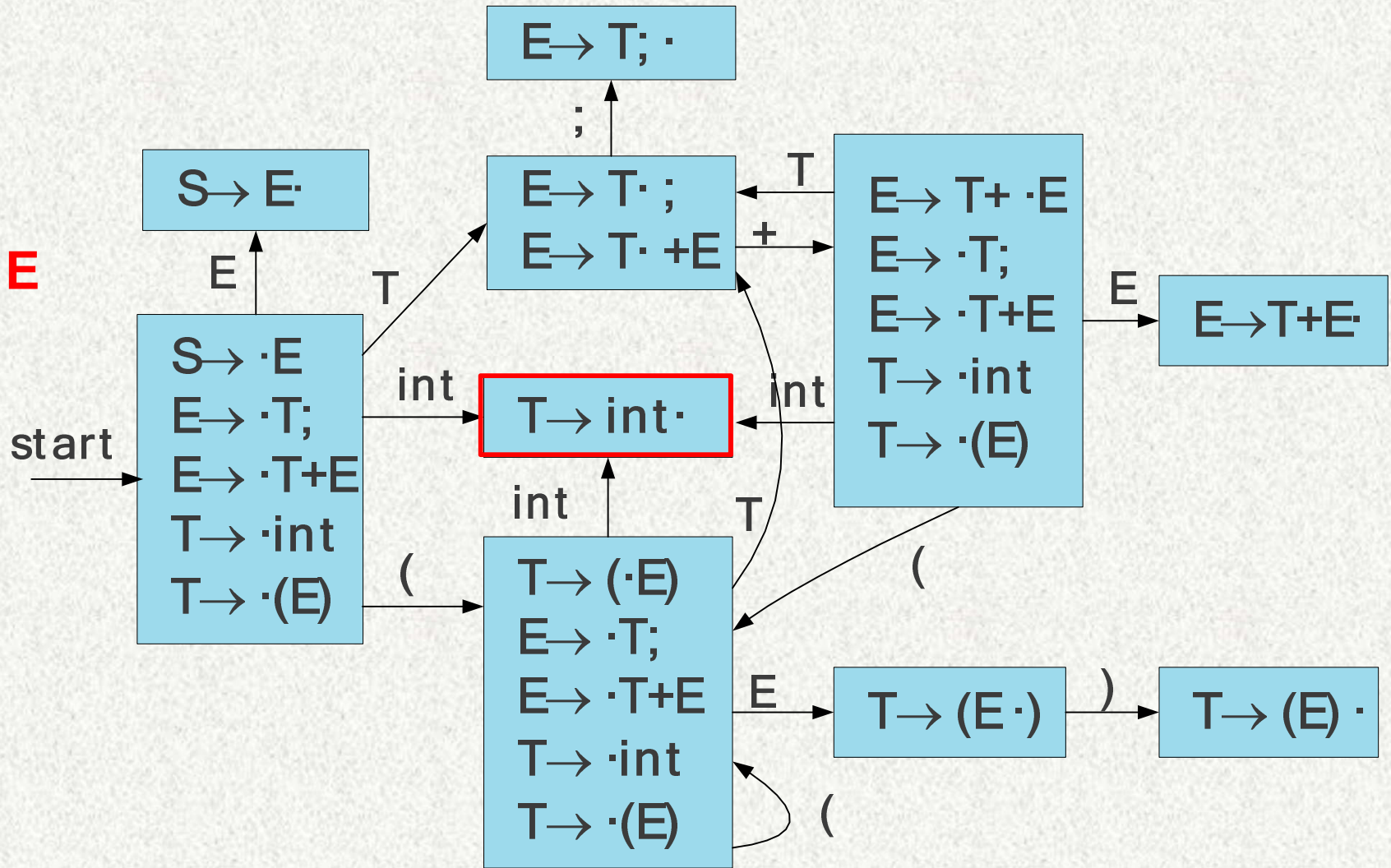
LR(0) Parsing

$S \rightarrow E$
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 $T \rightarrow (E)$



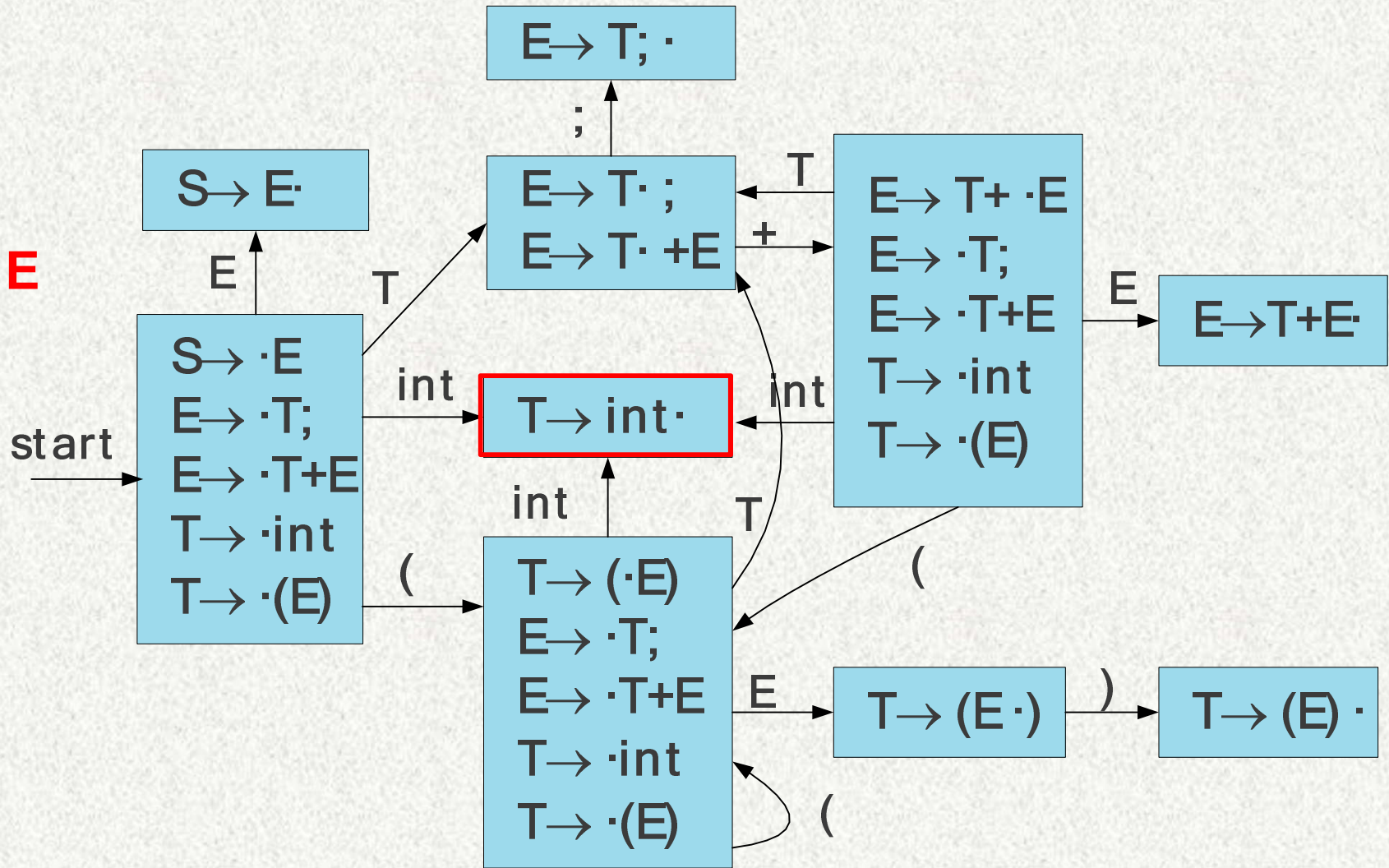
LR(0) Parsing

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 $T \rightarrow \text{int}$
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LR(0) Parsing

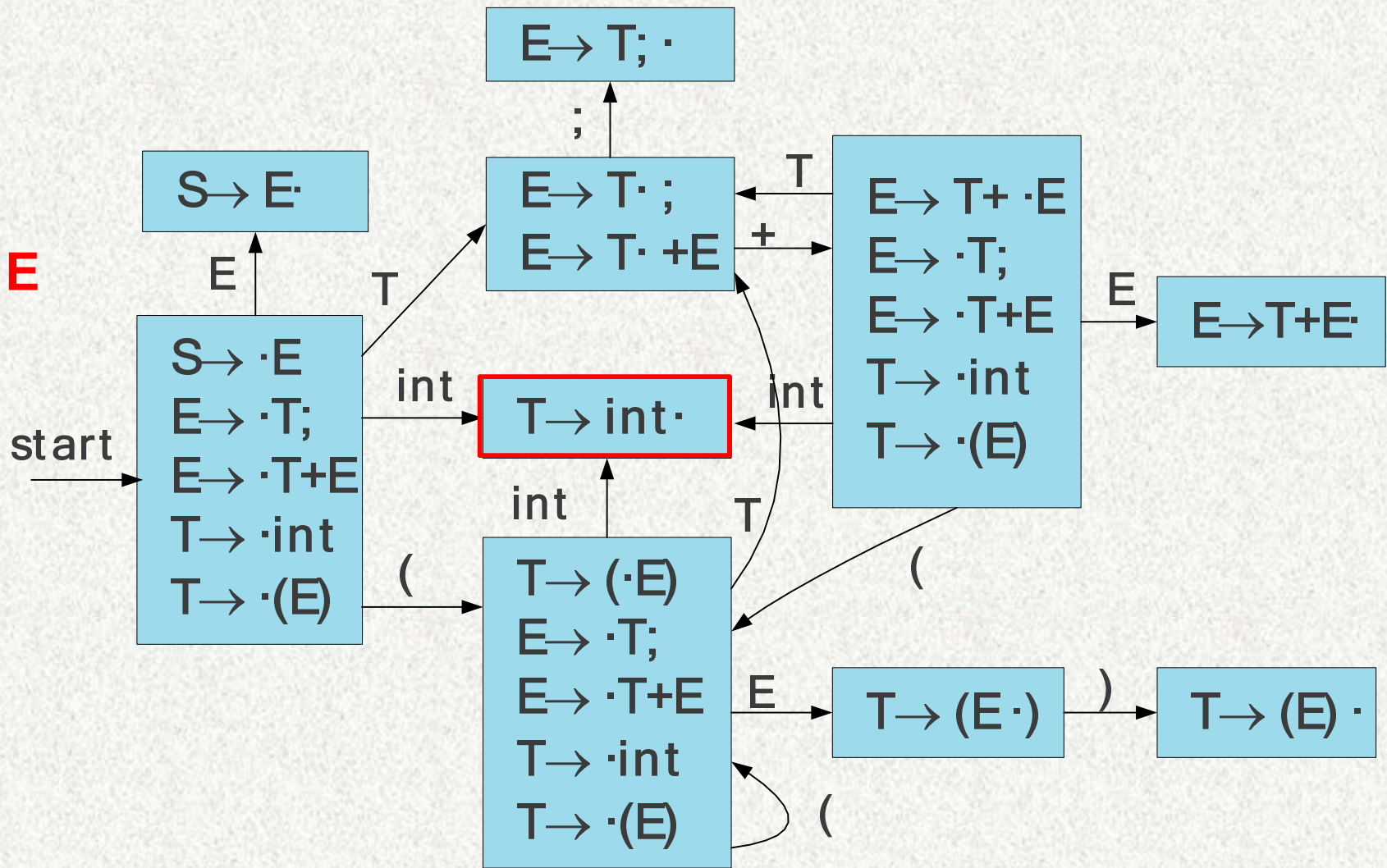
$S \rightarrow E$
 $E \rightarrow T;$
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+	(int	+	int	;)	;
---	---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
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 $T \rightarrow (E)$

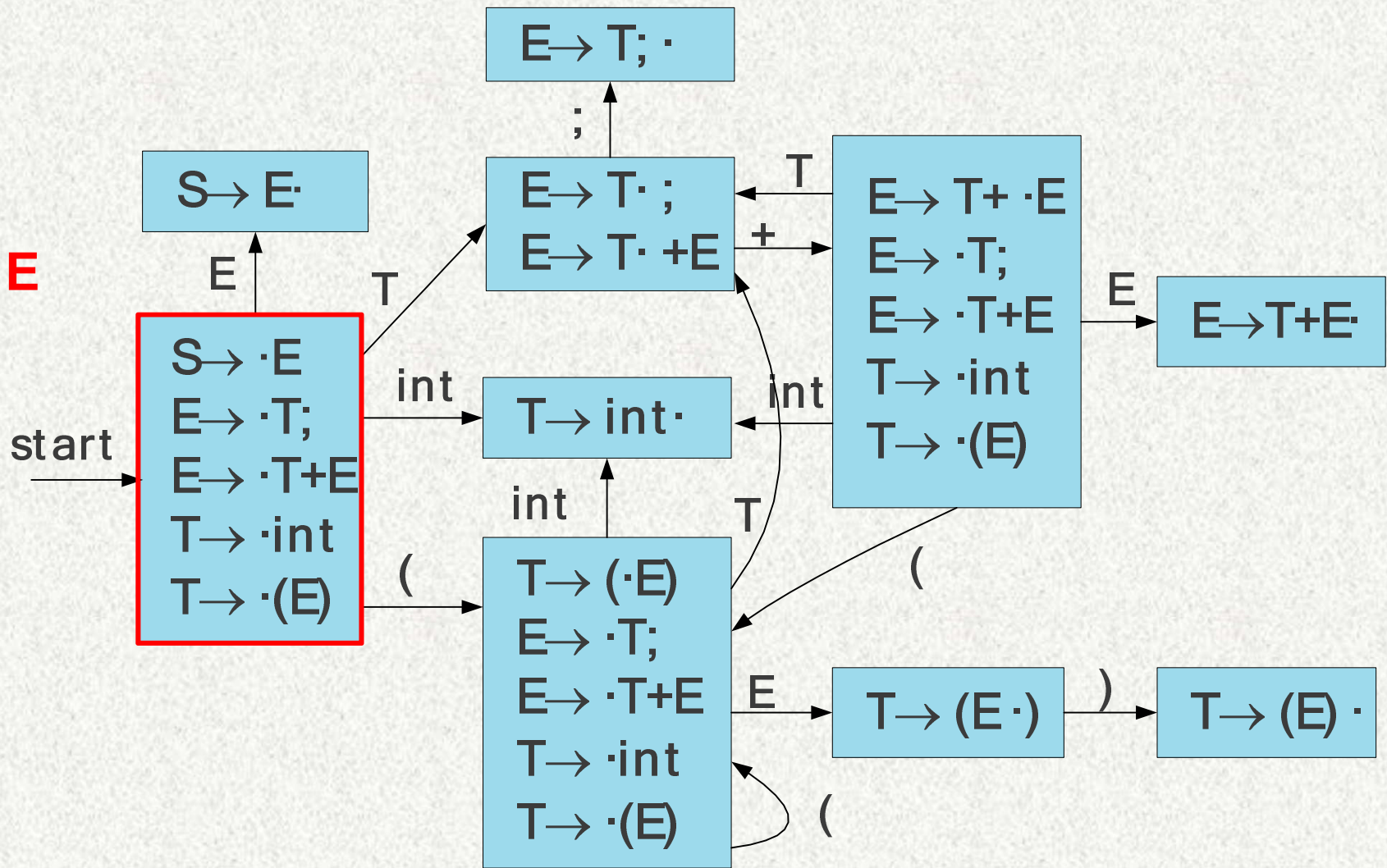


T

+	(int	+	int	;)	;
---	---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
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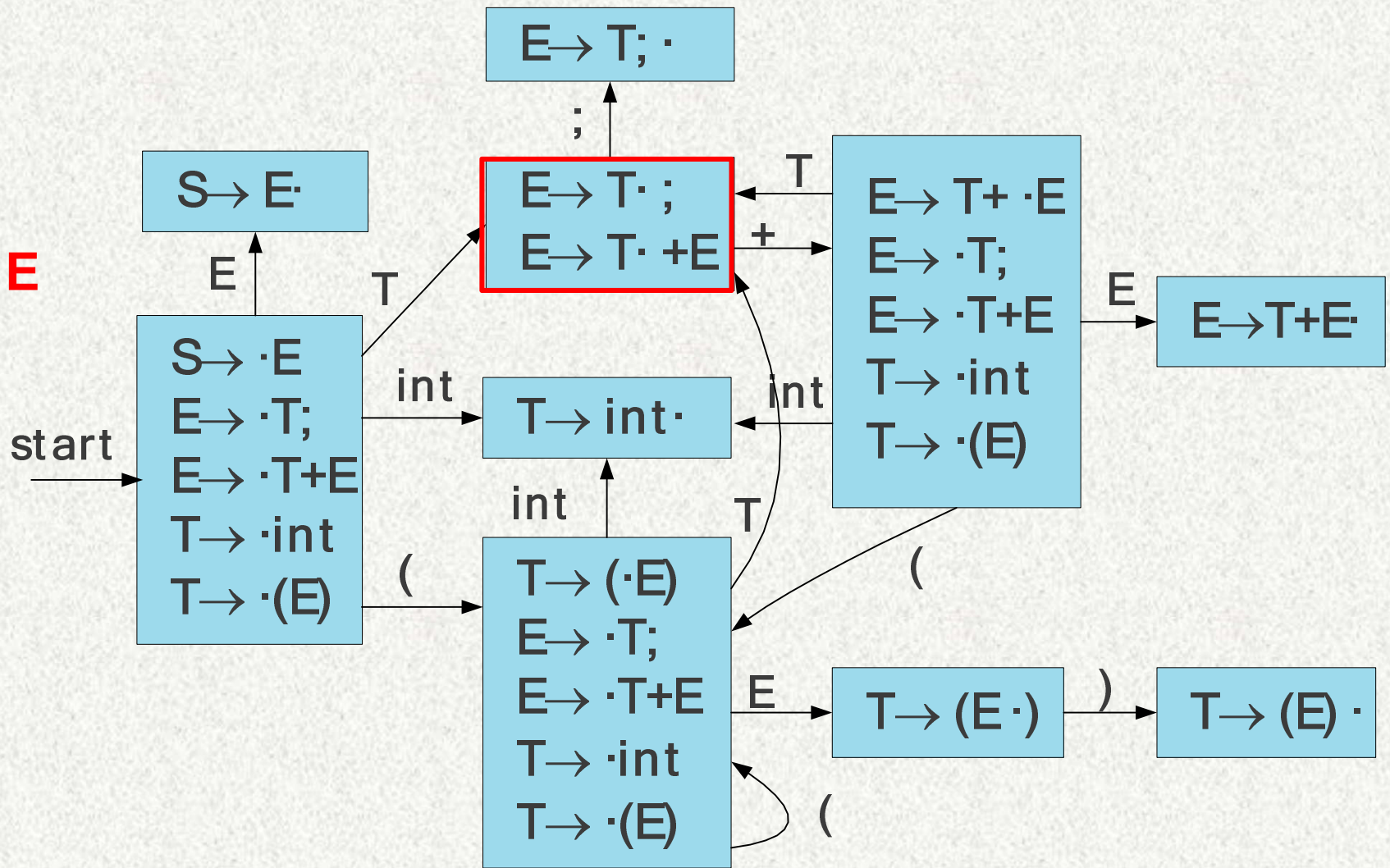


T

+	(int	+	int	;)	;
---	---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
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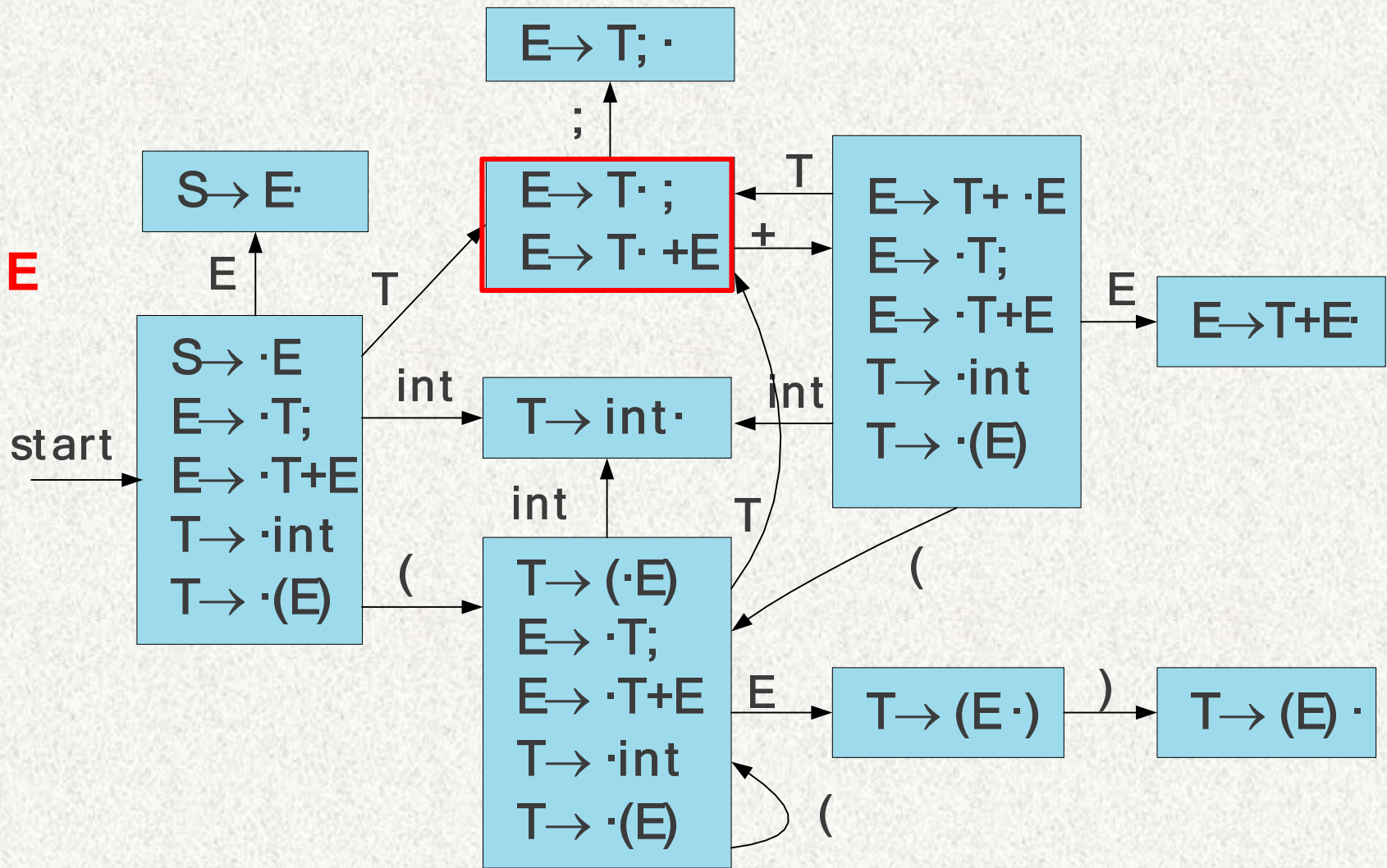


T

+	(int	+	int	;)	;
---	---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
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 $E \rightarrow T + E$
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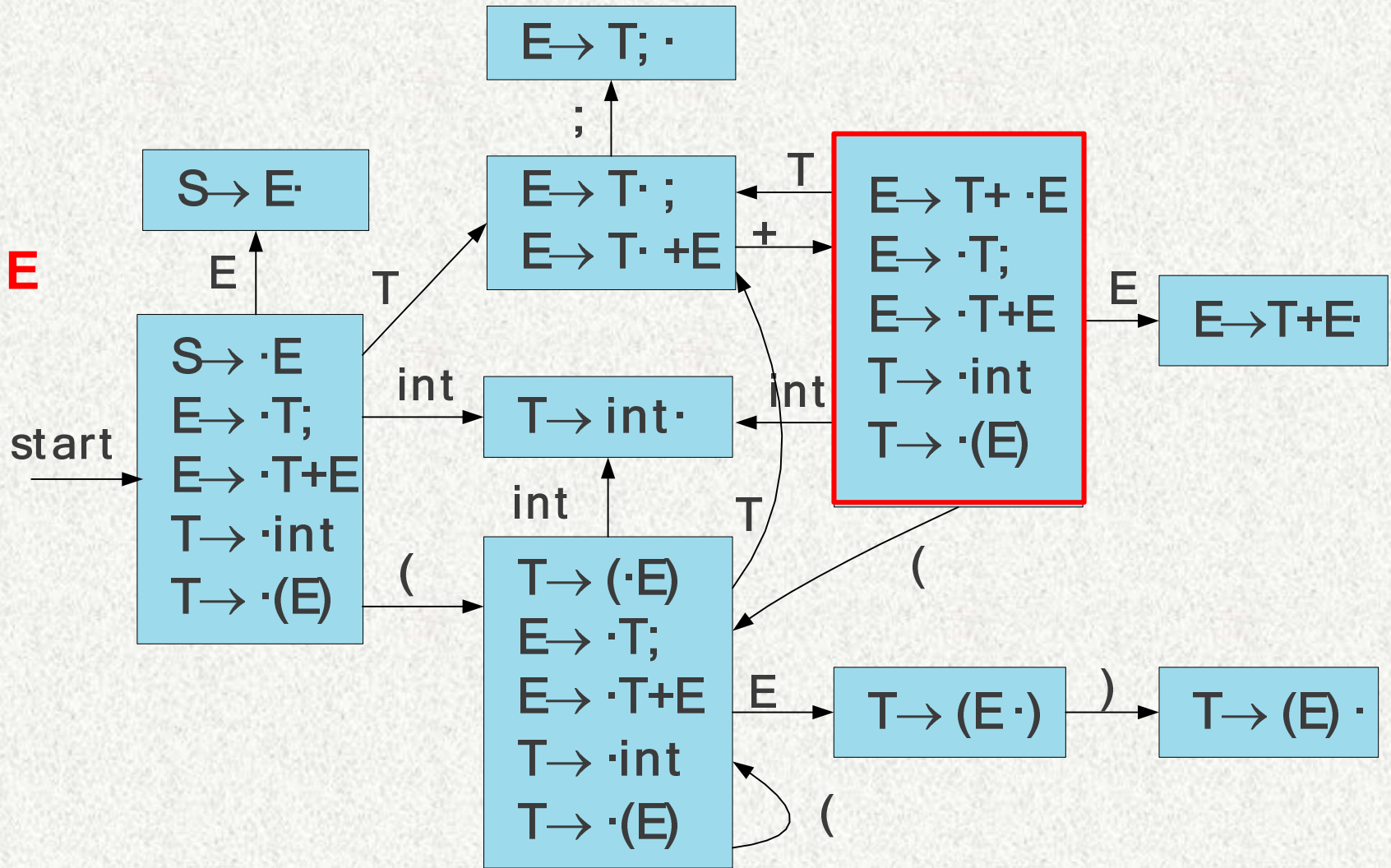


T	+
---	---

(int	+	int	;)	;
---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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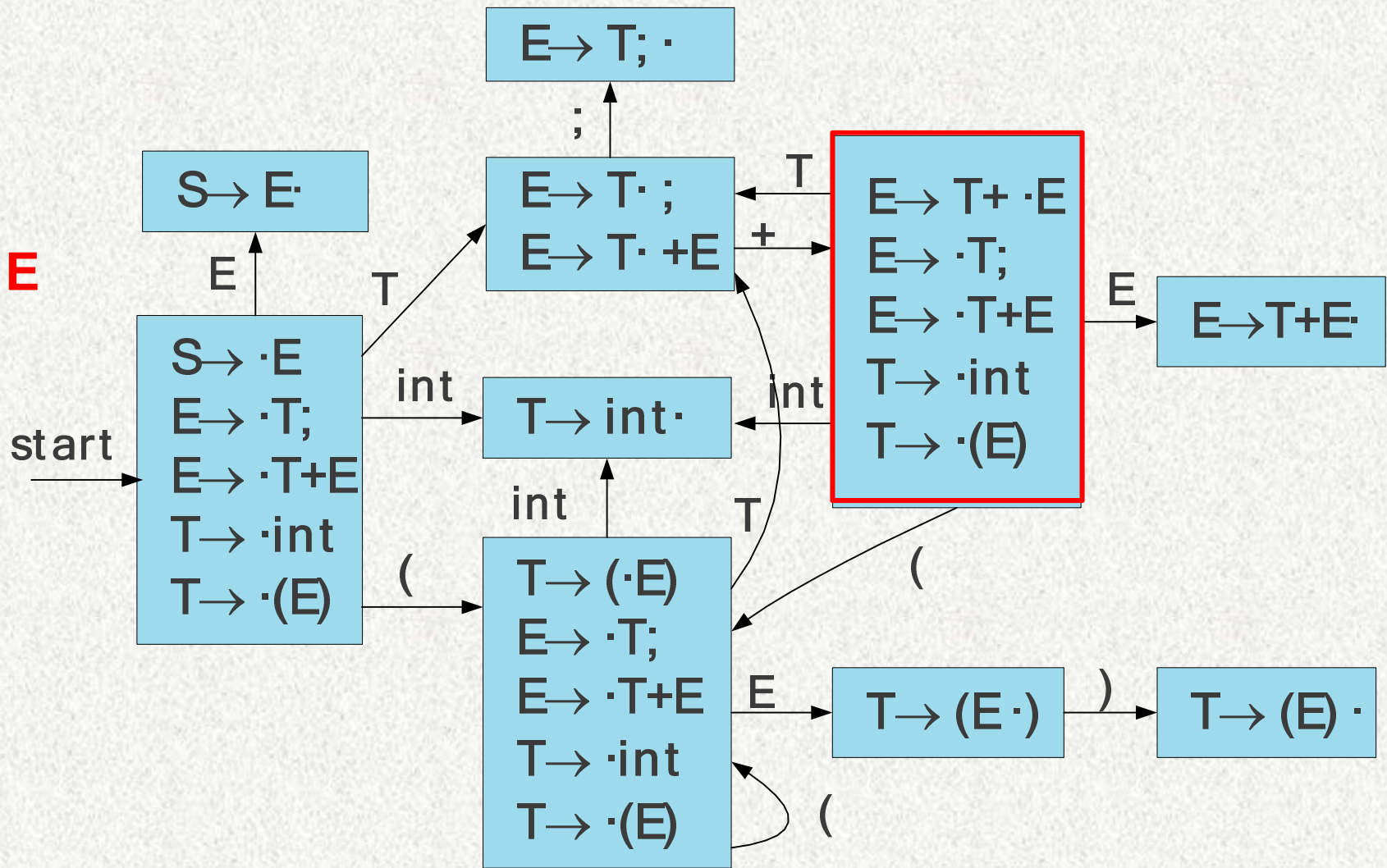


T	+
---	---

(int	+	int	;)	;
---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
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 $E \rightarrow T + E$
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 $T \rightarrow (E)$

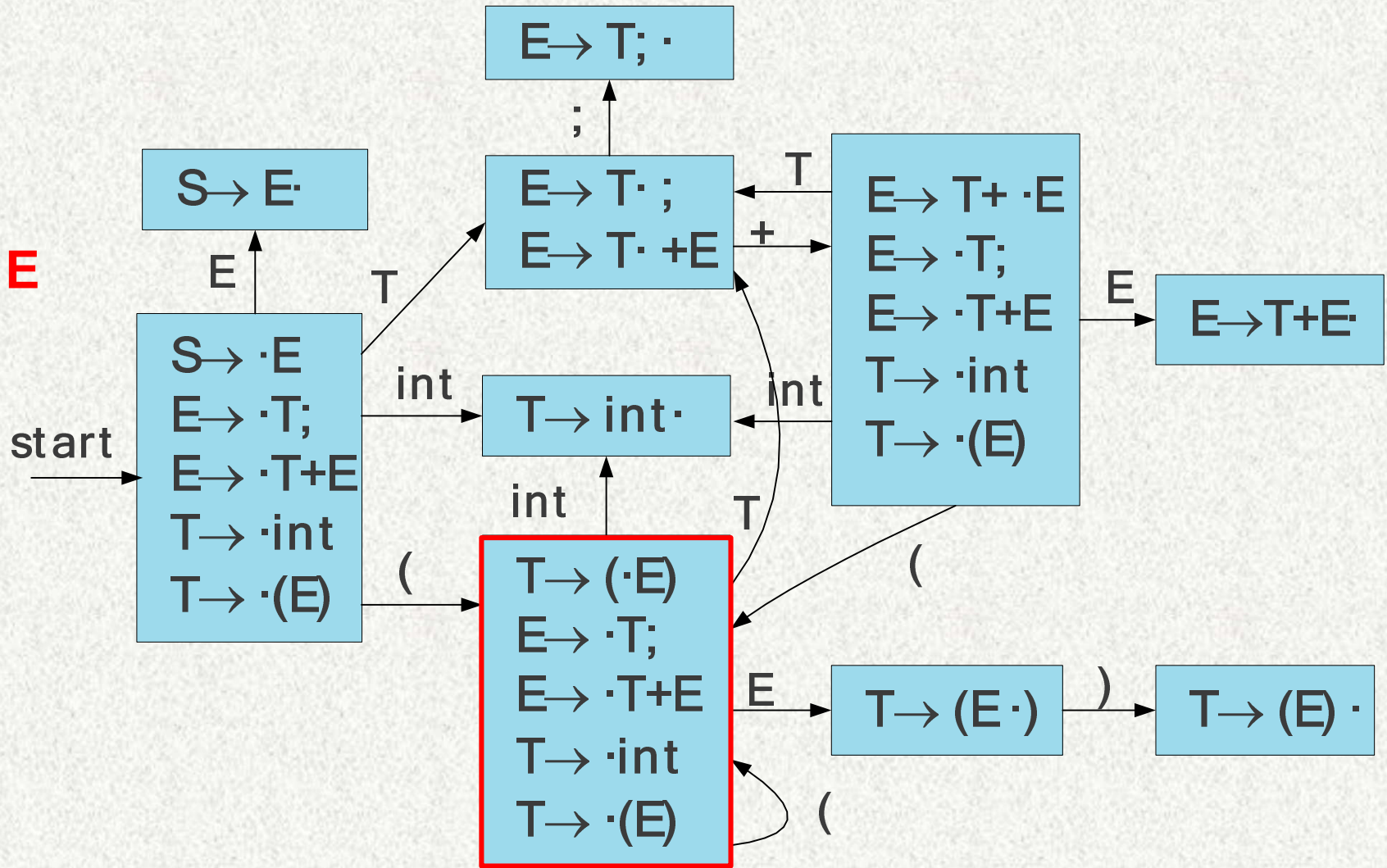


T	+	(
---	---	---

int	+	int	;)	;
-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

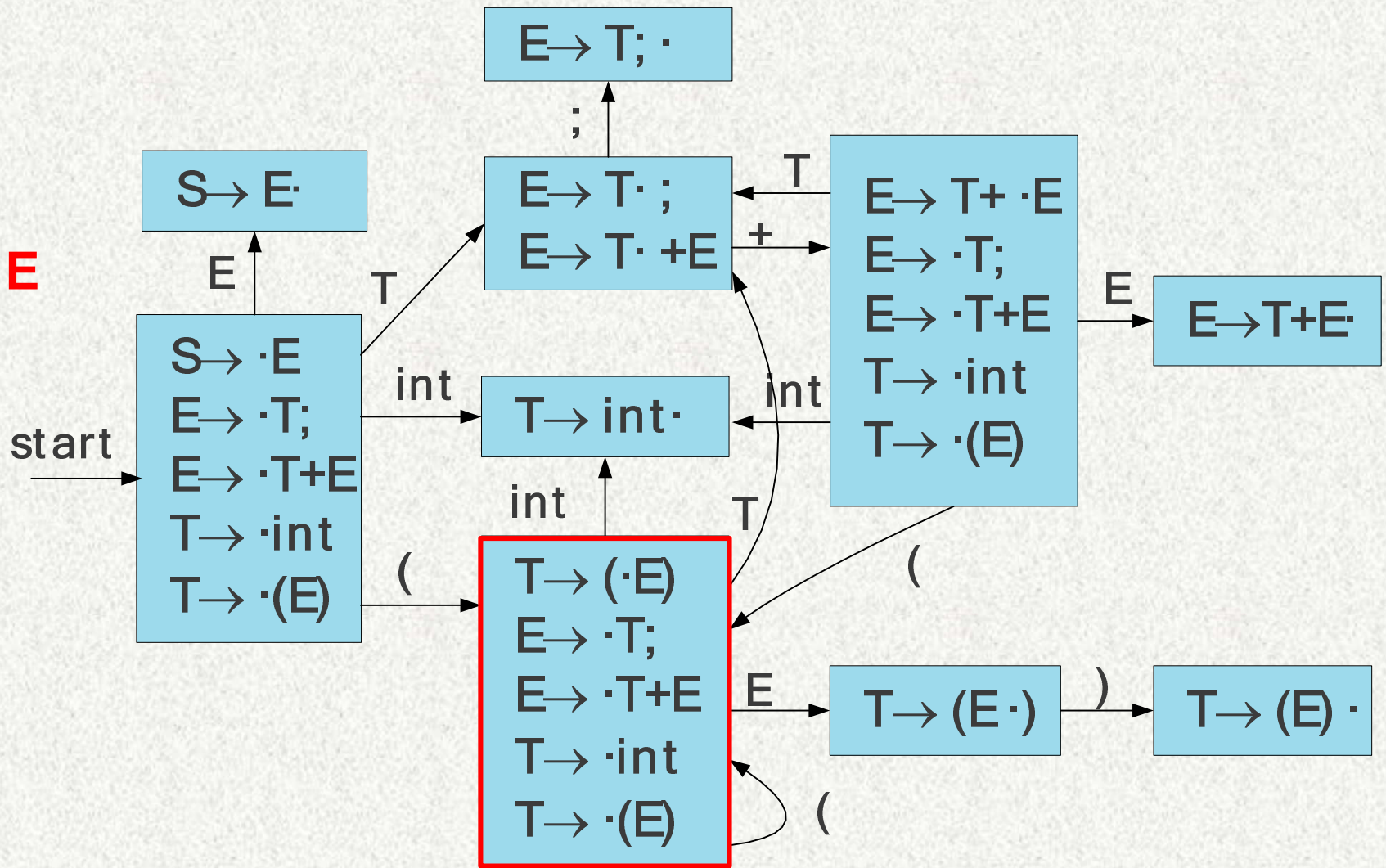


T	+	(
---	---	---

int	+	int	;)	;
-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
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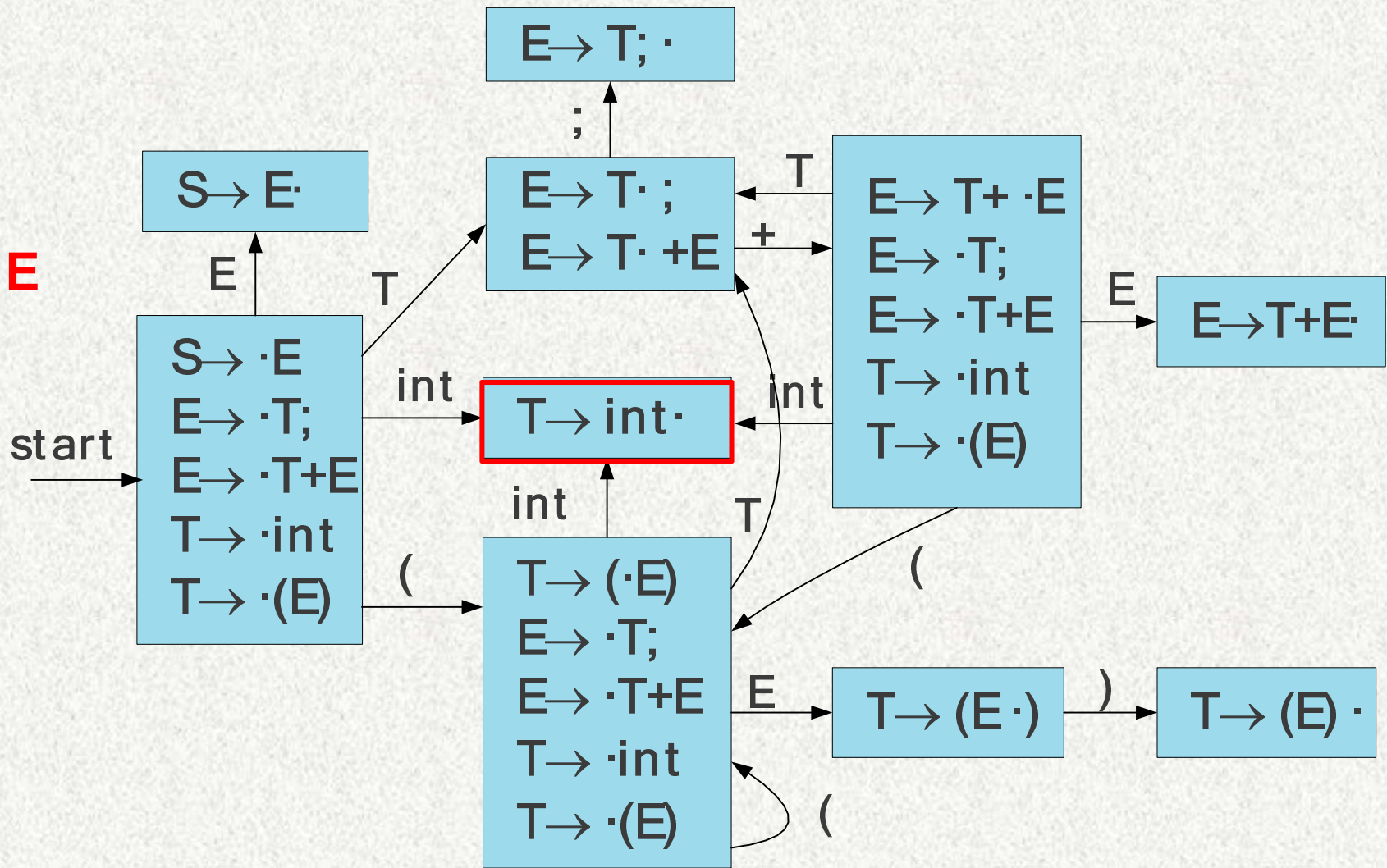


T	+	(int
---	---	---	-----

+	int	;)	;
---	-----	---	---	---

LR(0) Parsing

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 $E \rightarrow T;$
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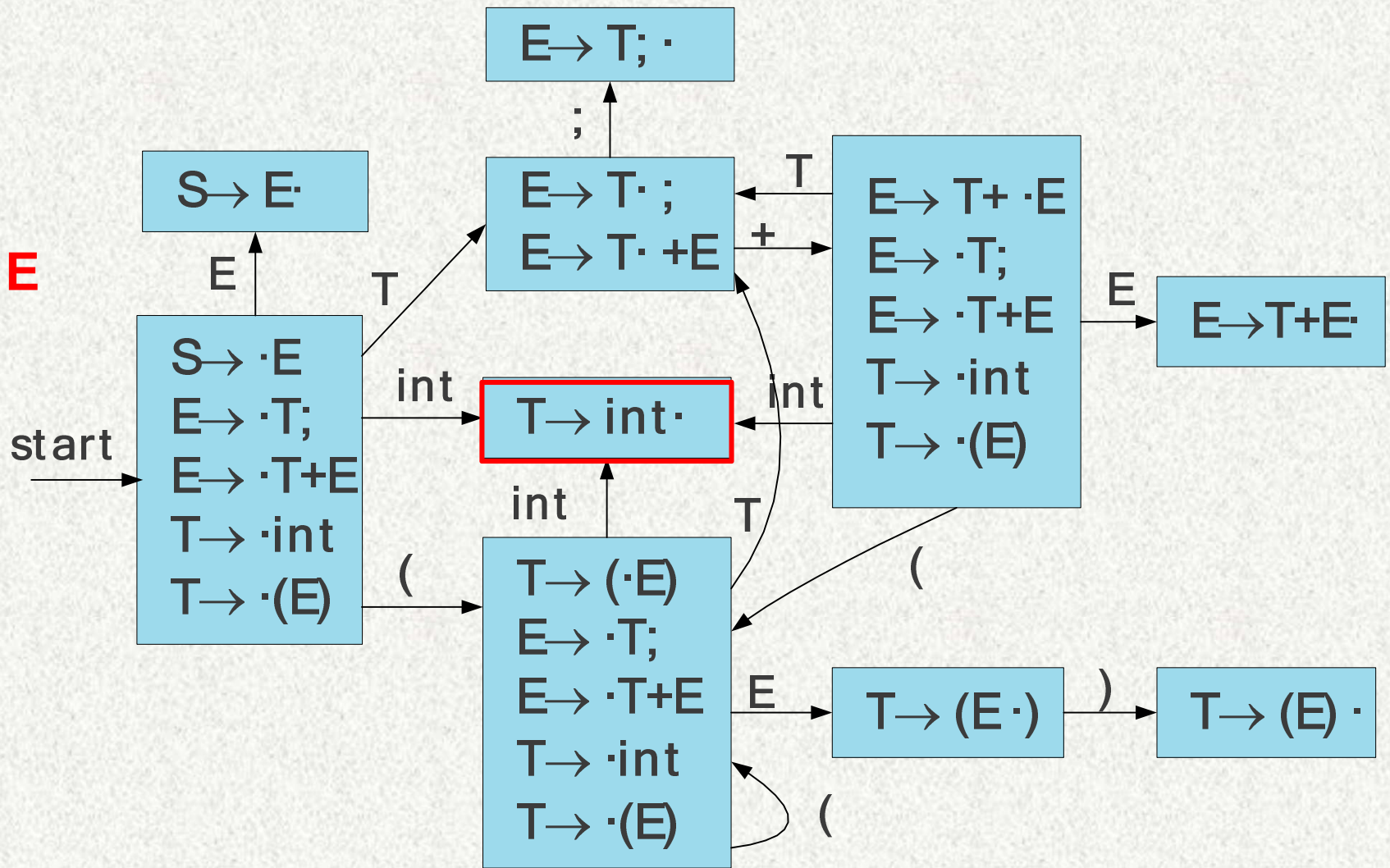


T	+	(int
---	---	---	-----

+	int	;)	;
---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
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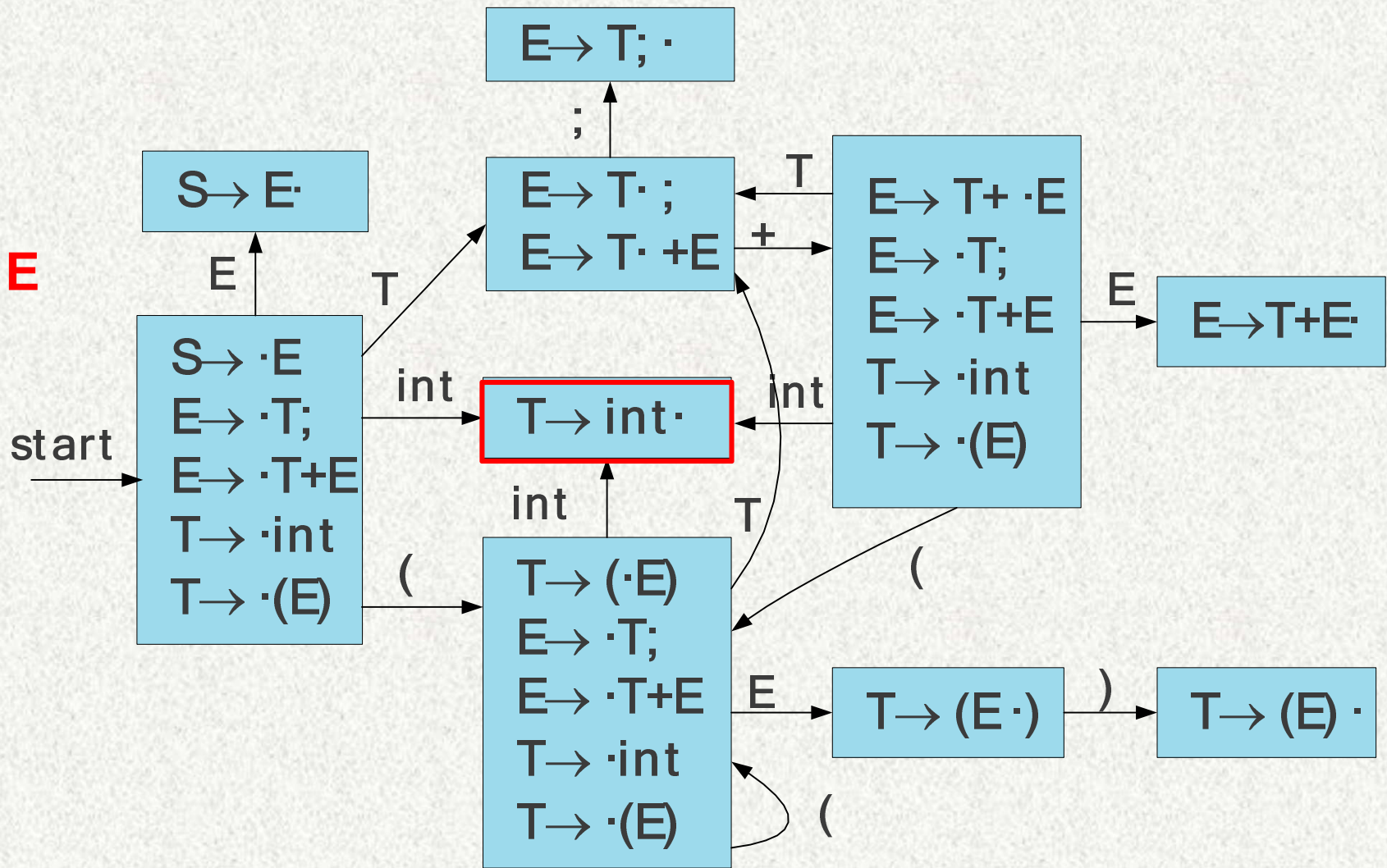


T	+	(
---	---	---	--

+	int	;)	;	
---	-----	---	---	---	--

LR(0) Parsing

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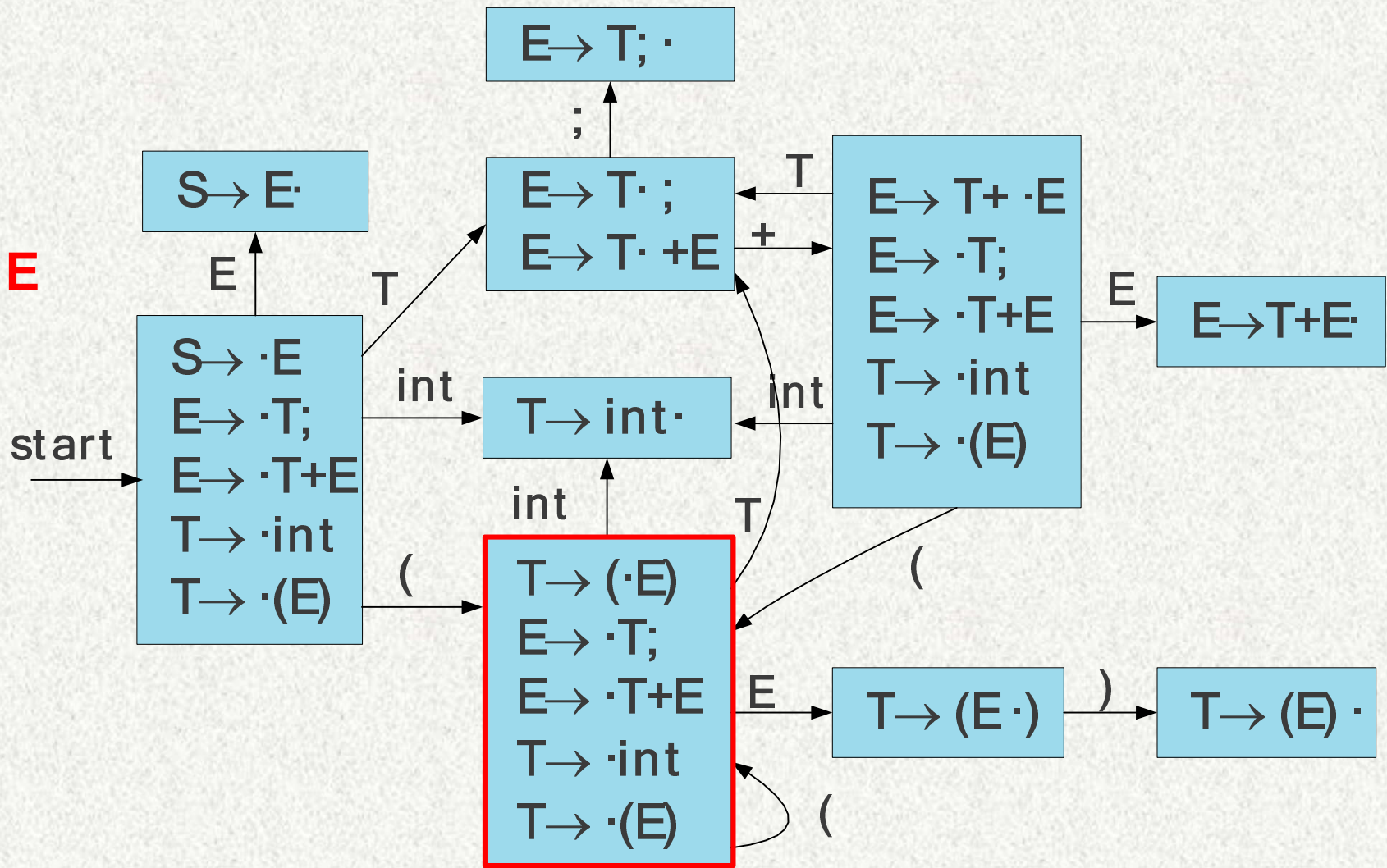


T	+	(T
---	---	---	---

+	int	;)	;
---	-----	---	---	---

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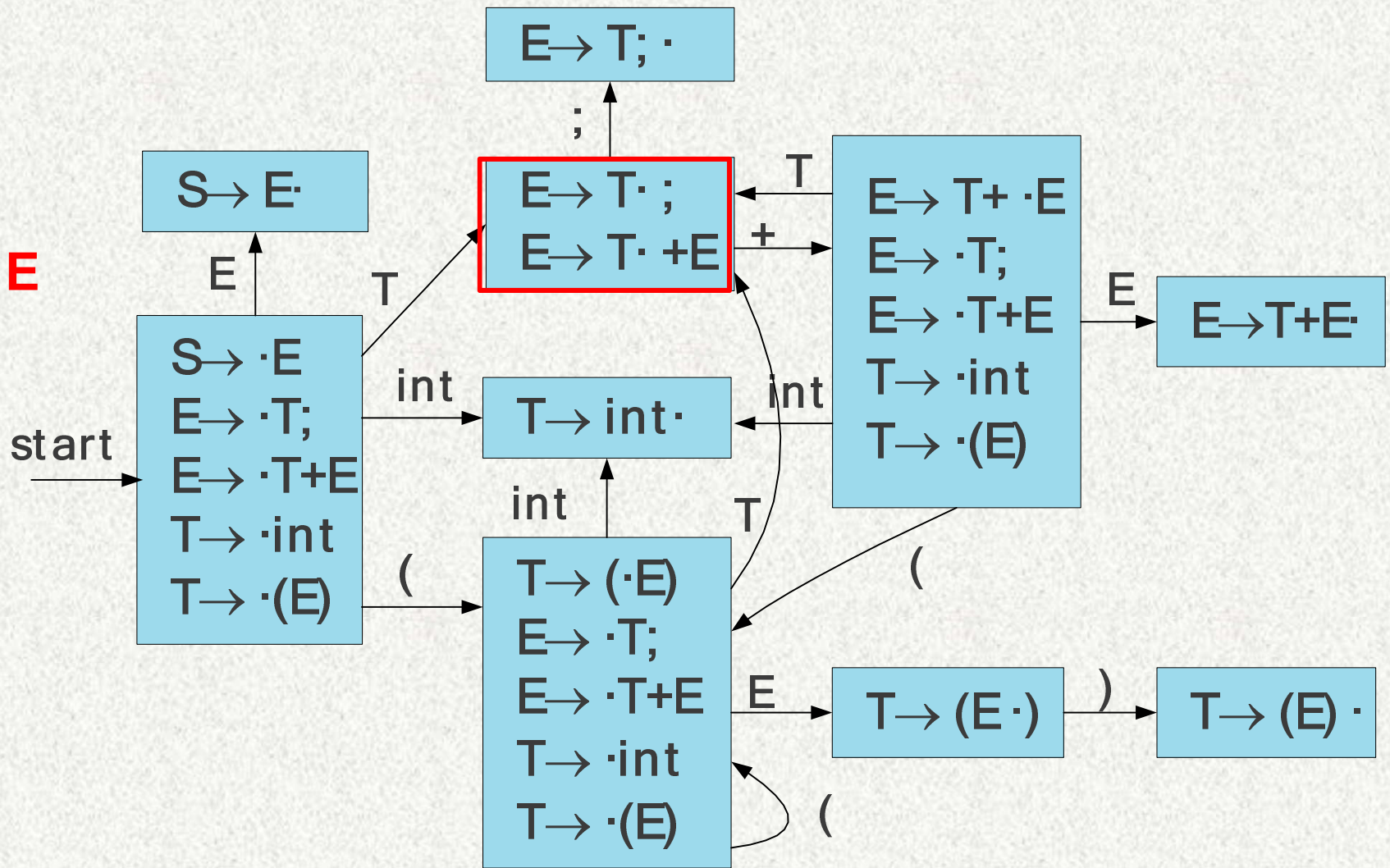


T	+	(T
---	---	---	---

+	int	;)	;	
---	-----	---	---	---	--

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$S \rightarrow E$
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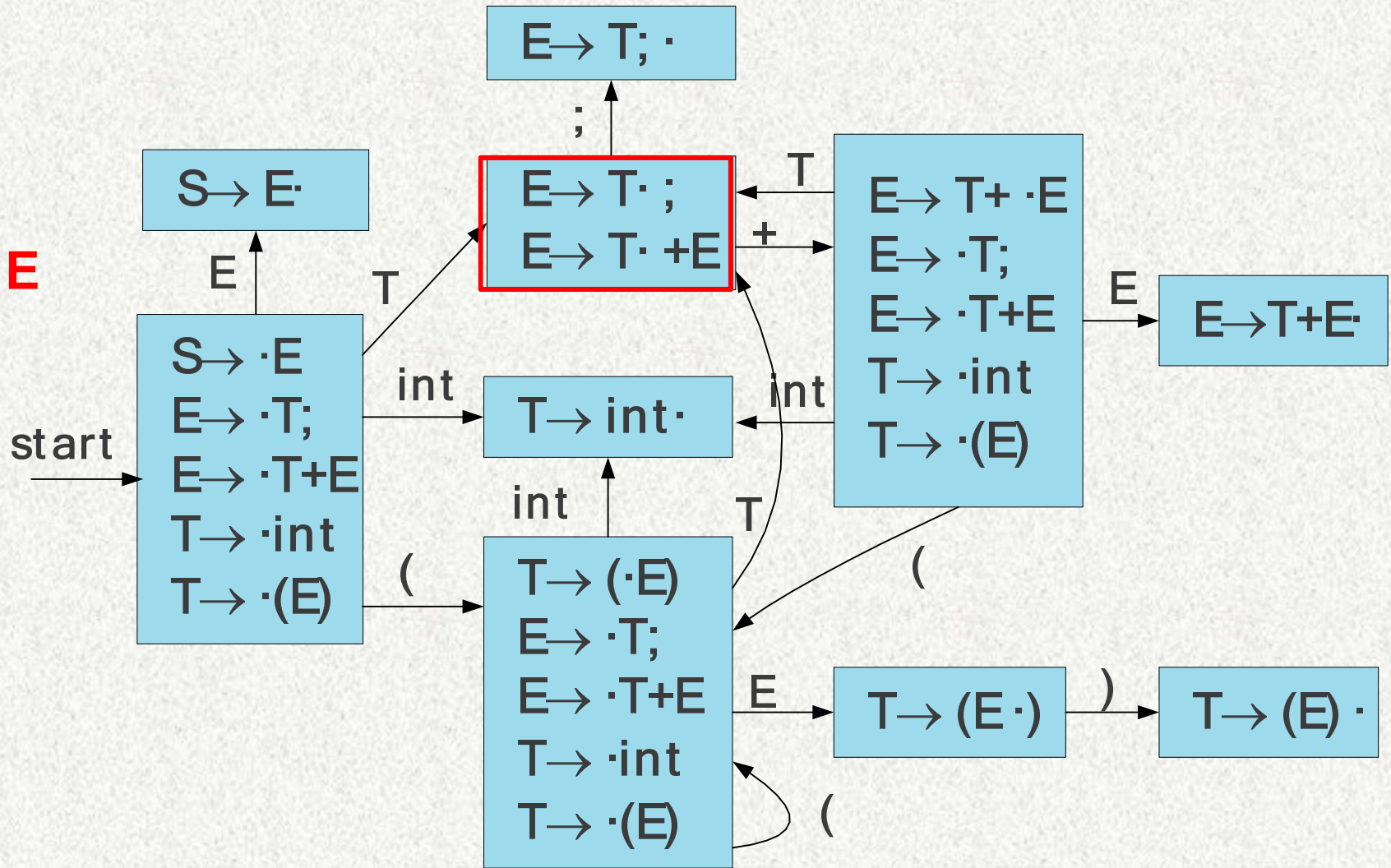


T	+	(T
---	---	---	---

+	int	;)	;	
---	-----	---	---	---	--

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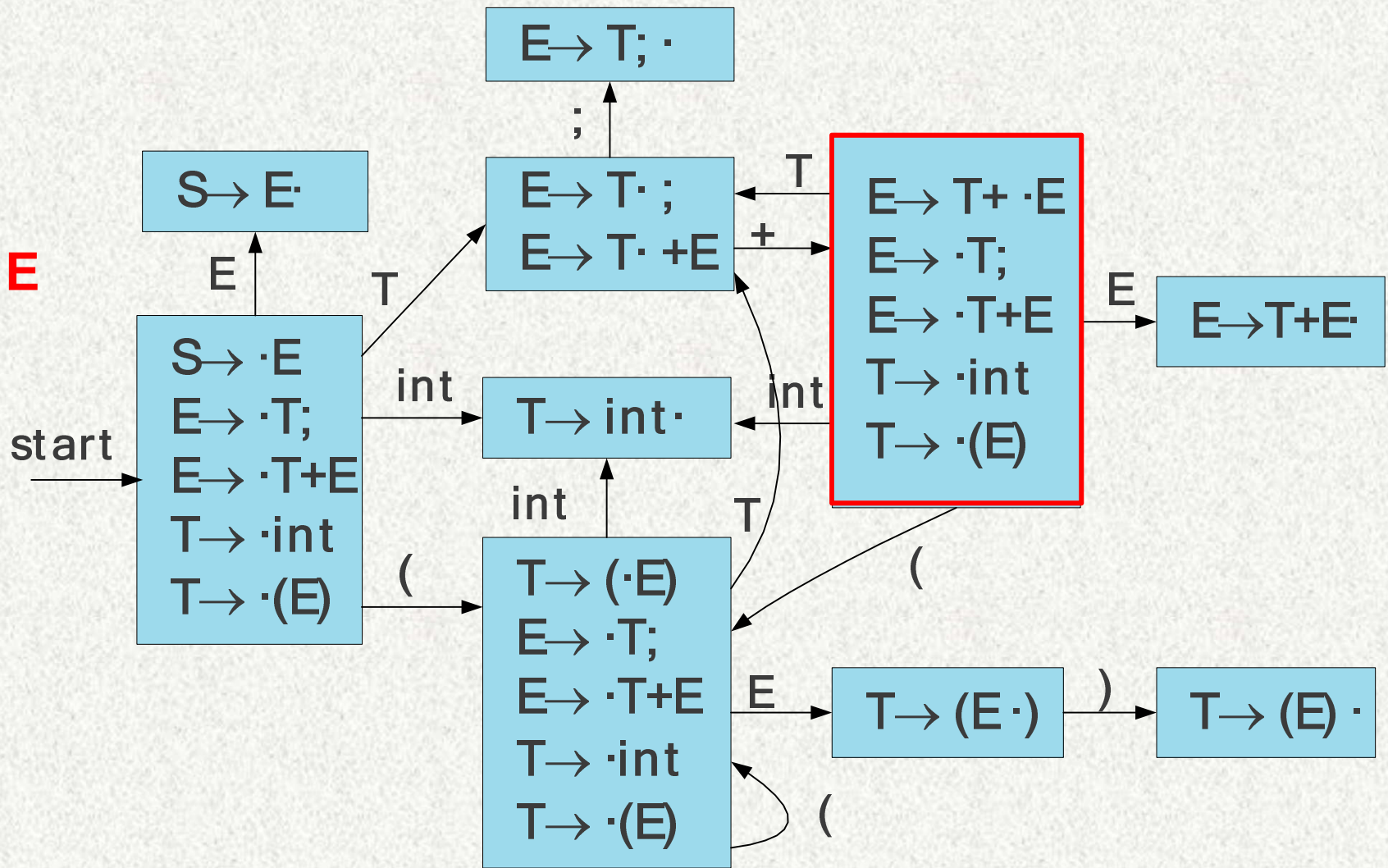


T	+	(T	+
---	---	---	---	---

int	;)	;
-----	---	---	---

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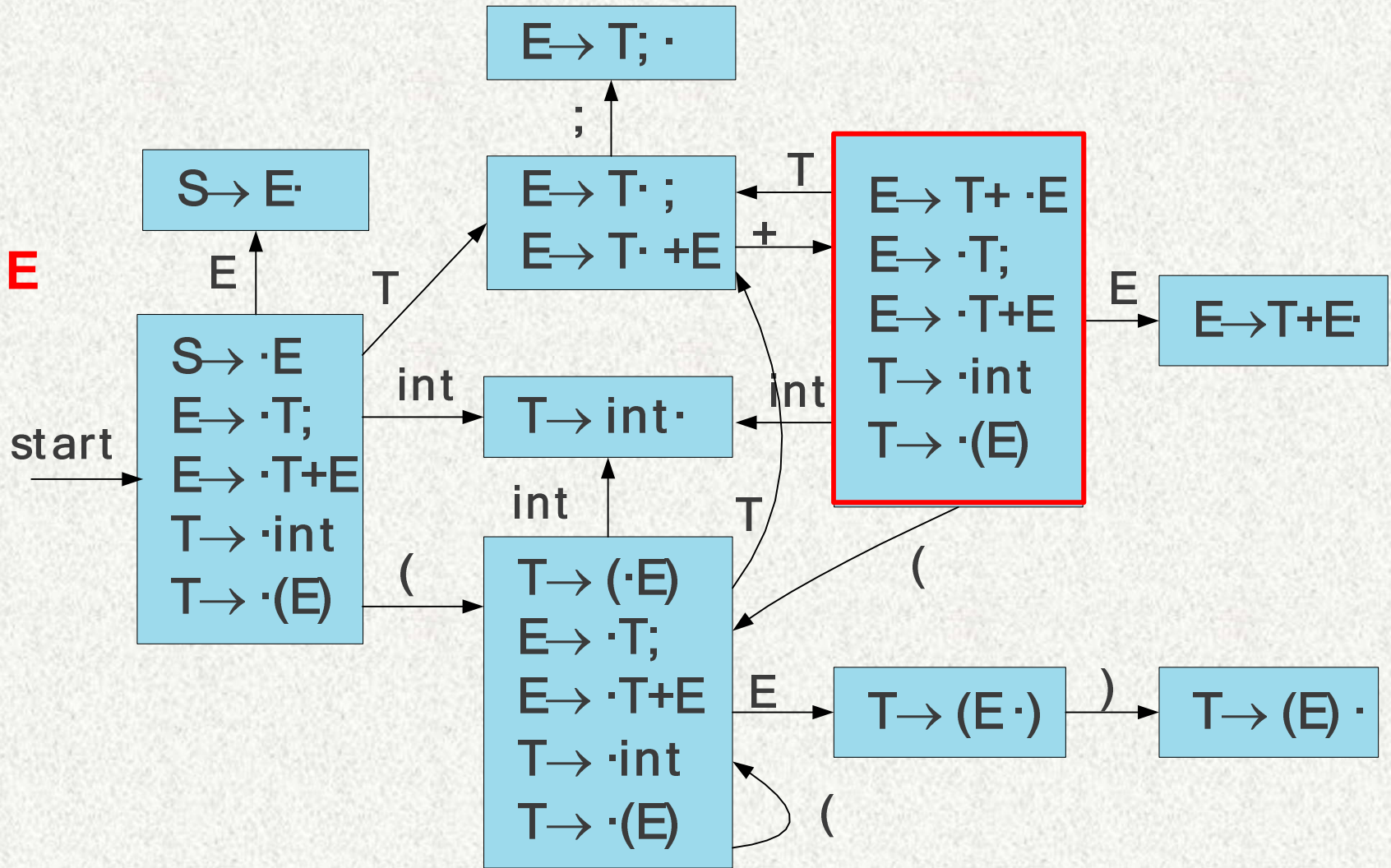


T	+	(T	+
---	---	---	---	---

int	;)	;
-----	---	---	---

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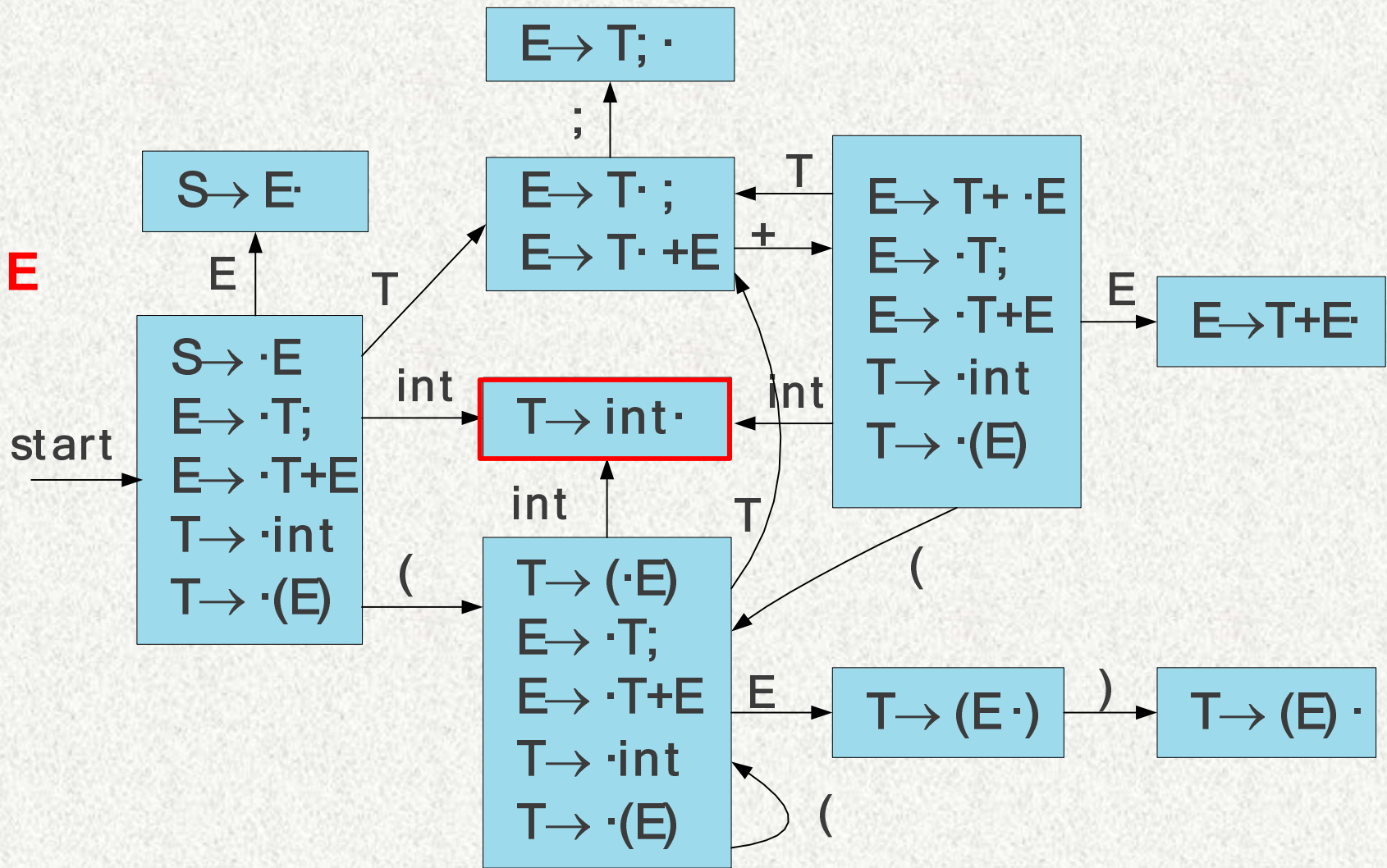


T	+	(T	+	int
---	---	---	---	---	-----

;)	;
---	---	---

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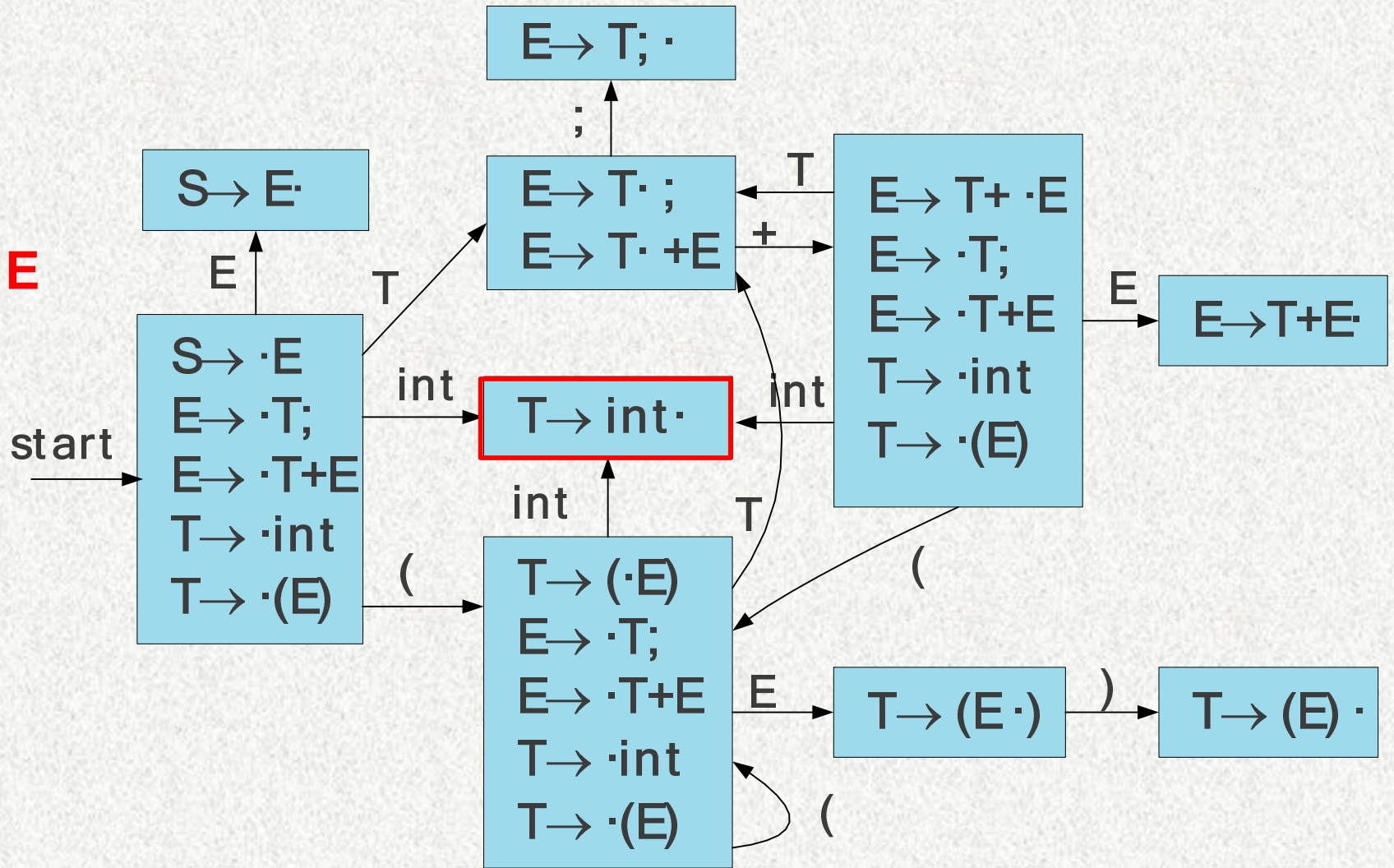


T	+	(T	+	int
---	---	---	---	---	-----

;)	;
---	---	---

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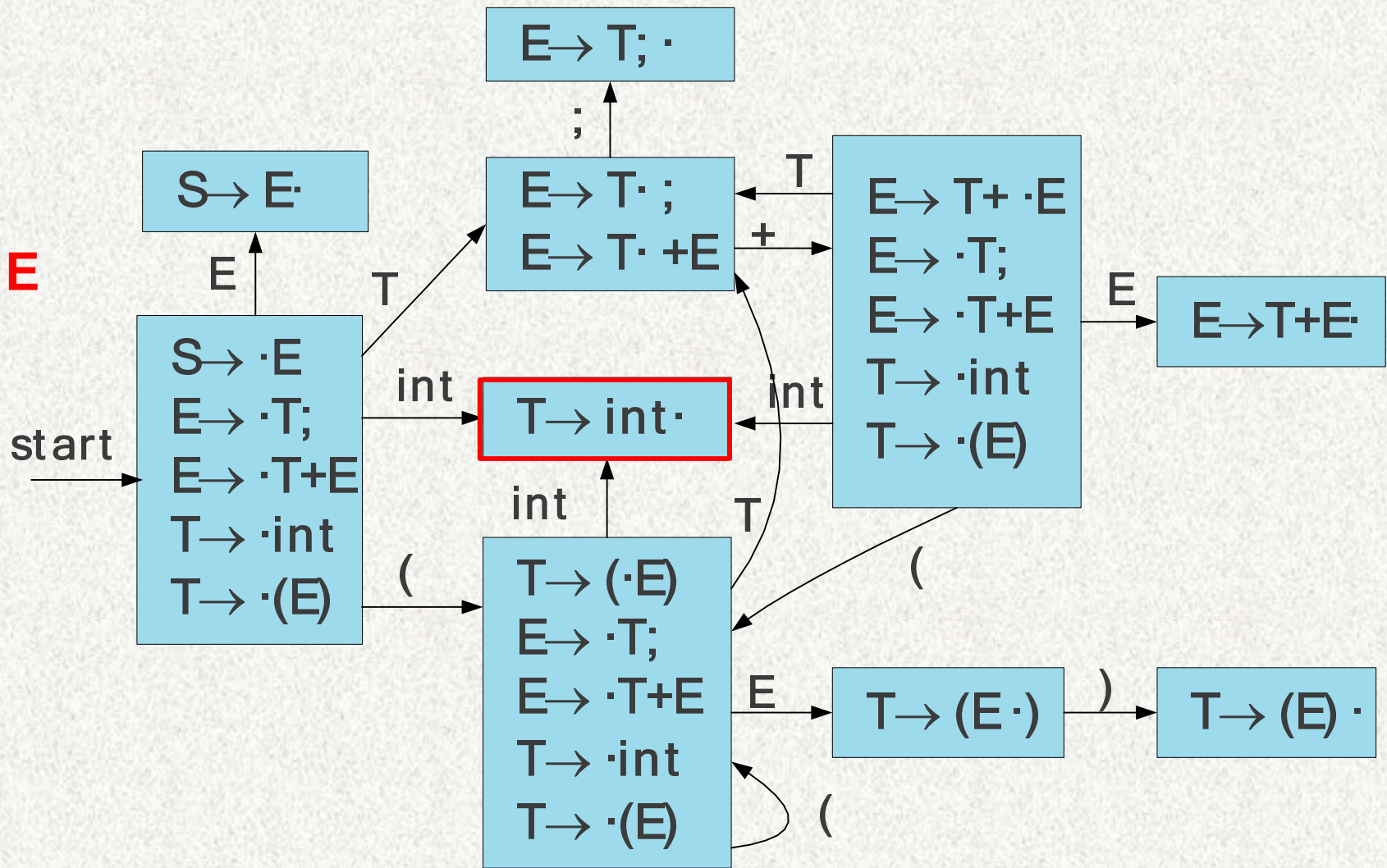


T	+	(T	+
---	---	---	---	---

;)	;
---	---	---

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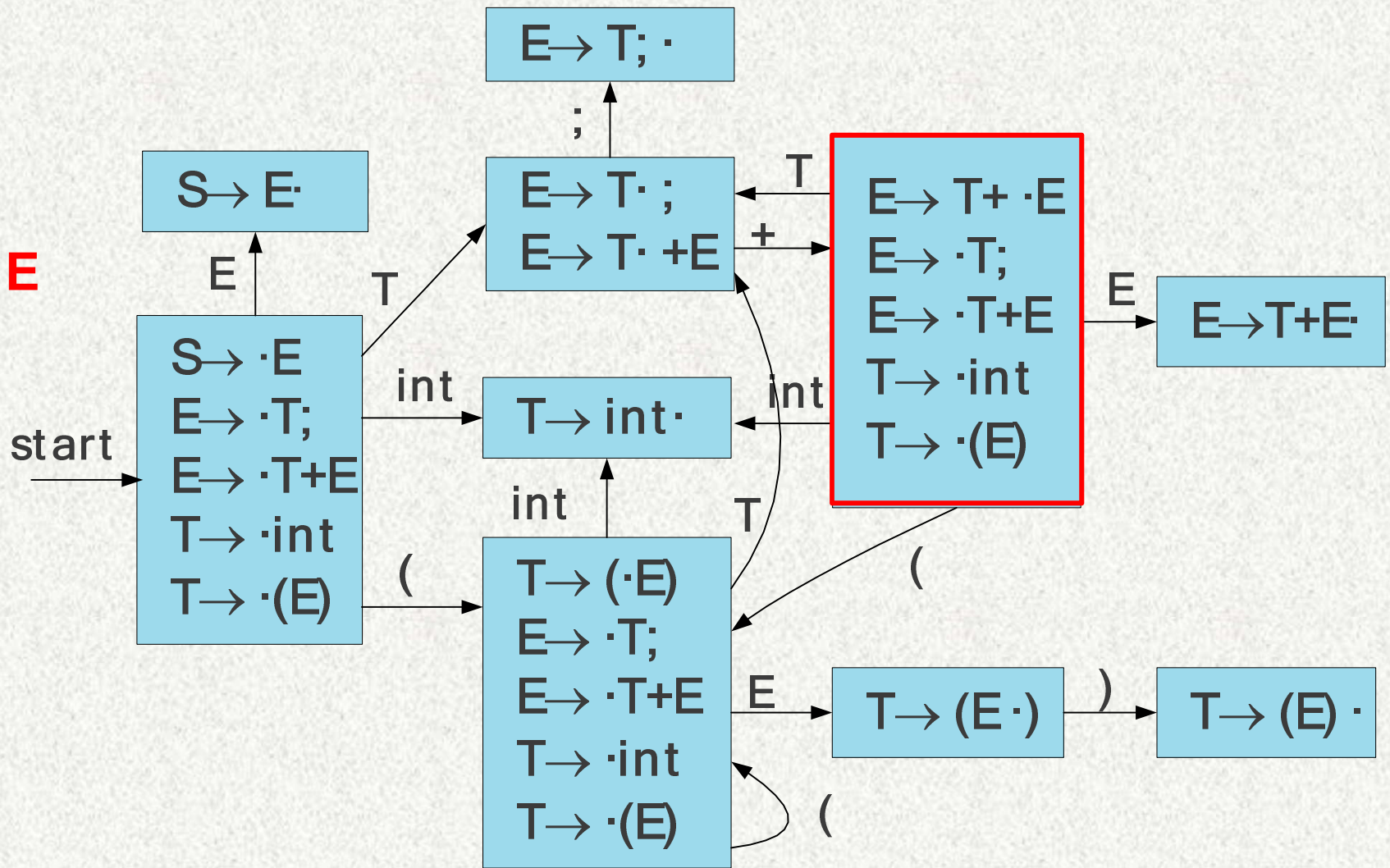


T	+	(T	+	T
---	---	---	---	---	---

;)	;
---	---	---

LR(0) Parsing

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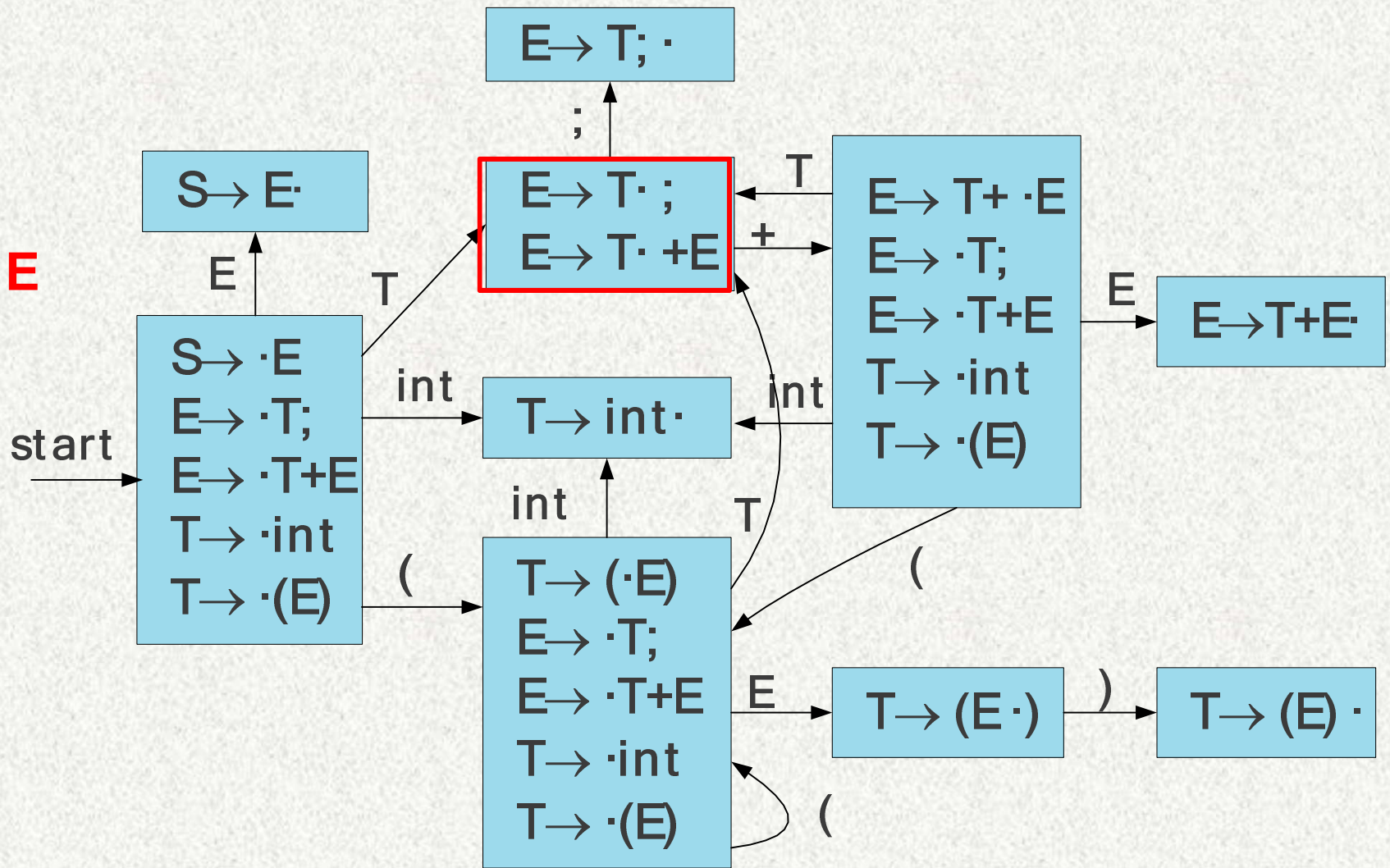


T	+	(T	+	T
---	---	---	---	---	---

;)	;
---	---	---

LR(0) Parsing

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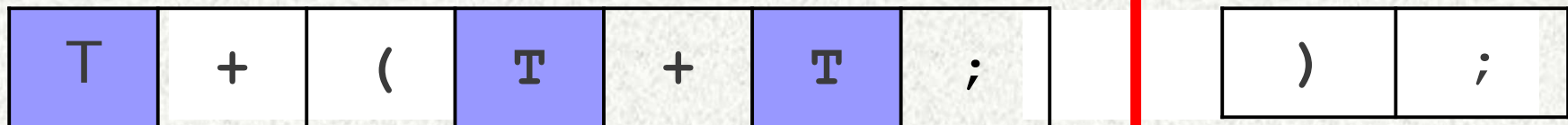
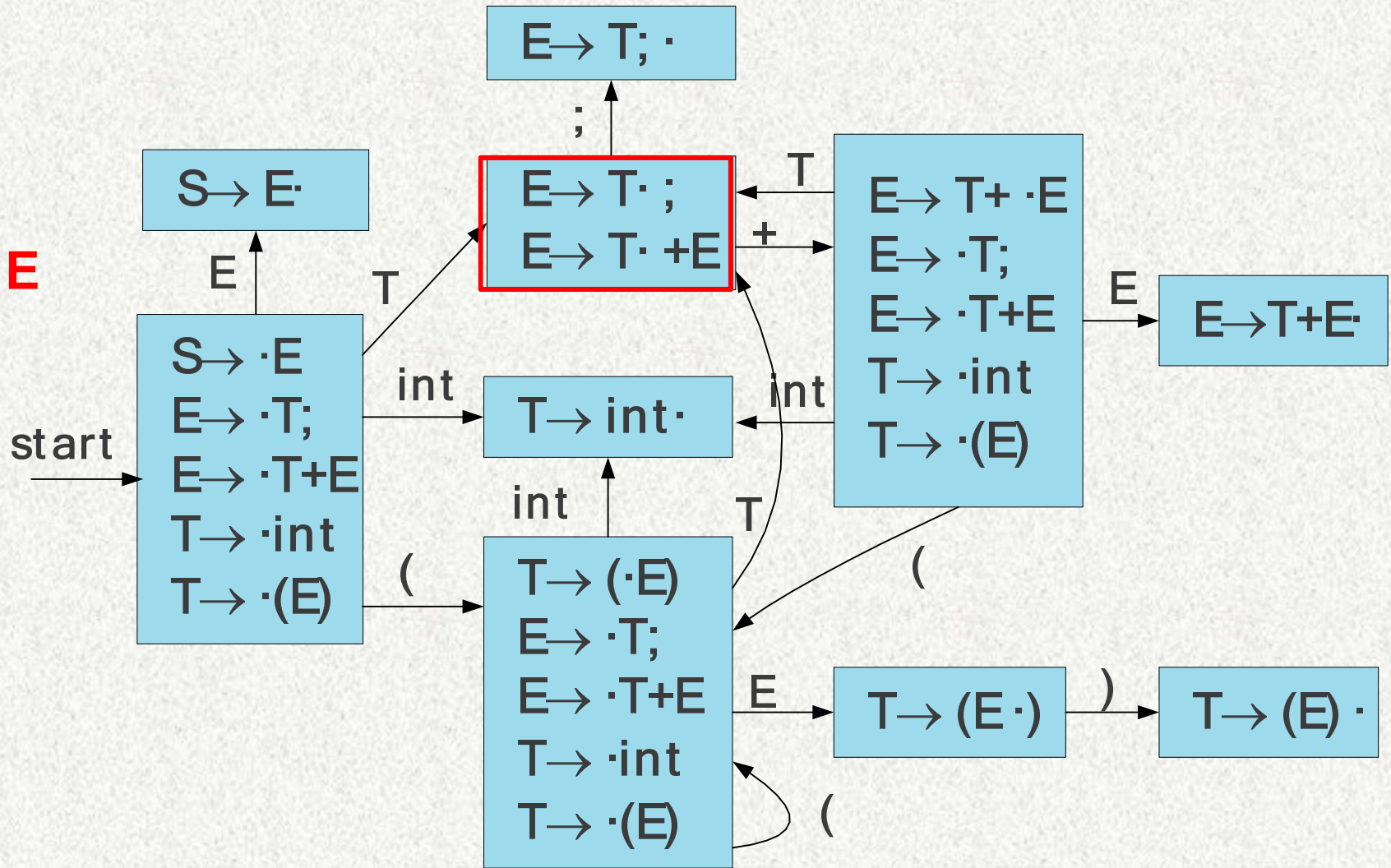


T	+	(T	+	T
---	---	---	---	---	---

;)	;
---	---	---

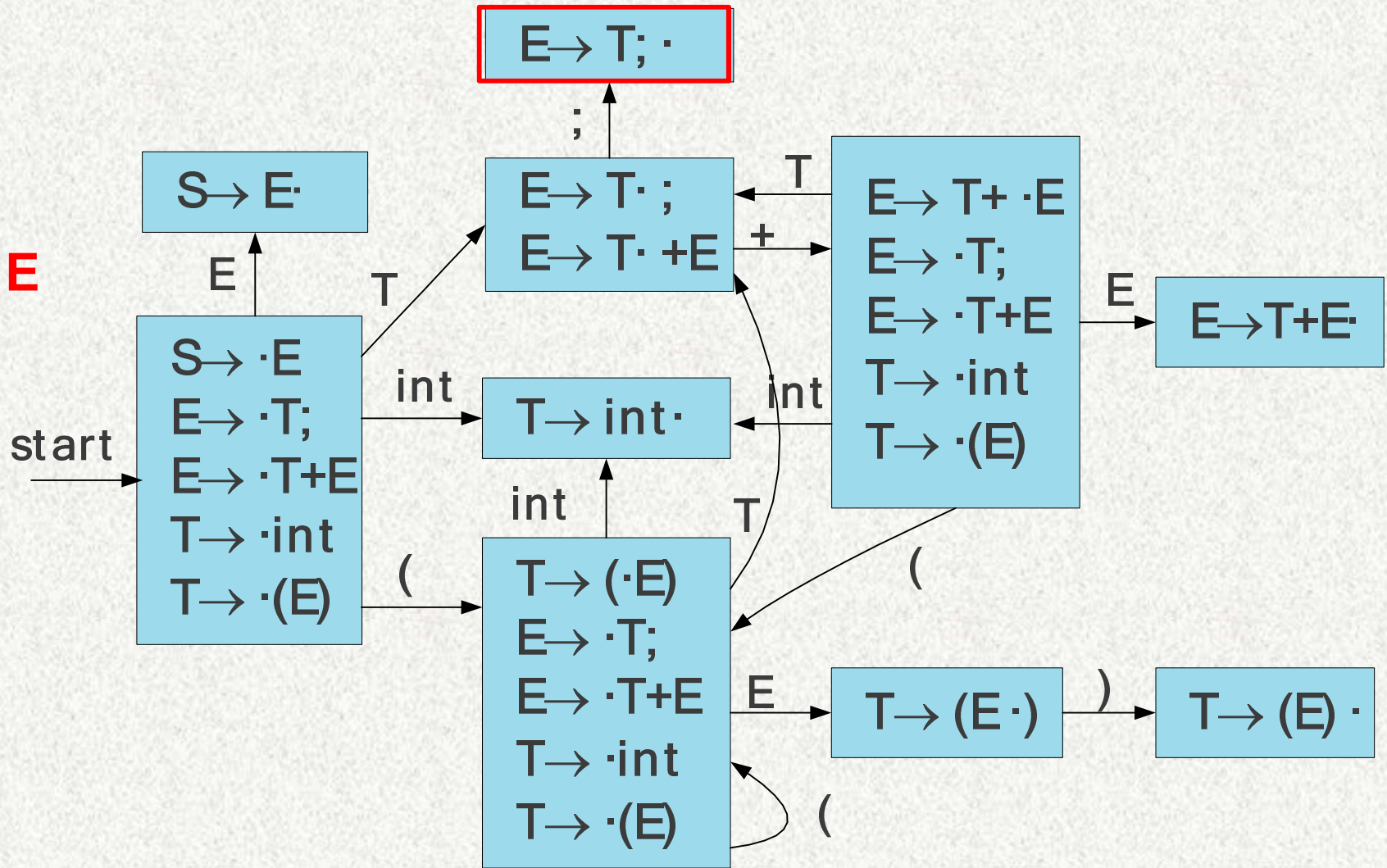
LR(0) Parsing

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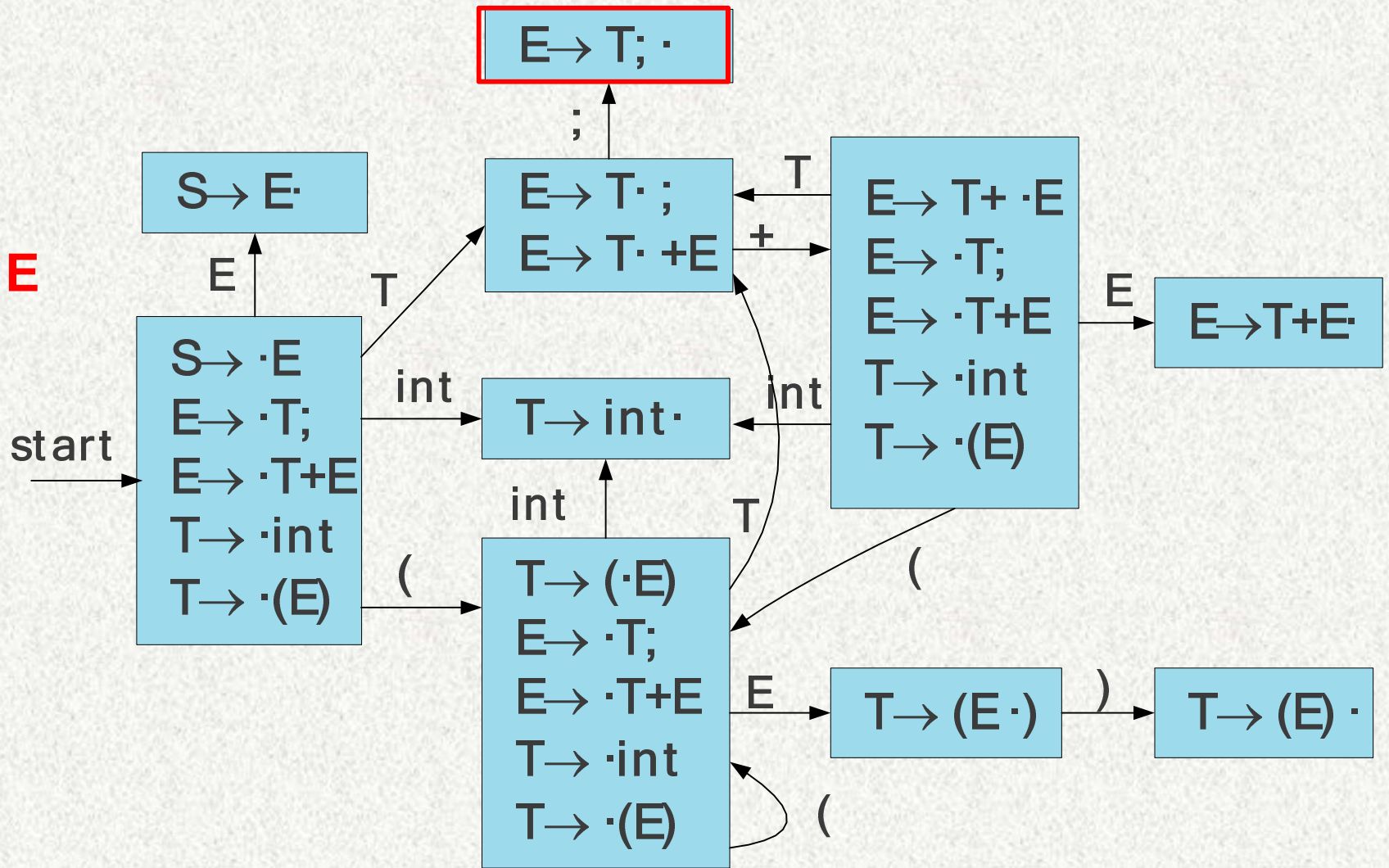


T	+	(T	+
---	---	---	---	---

)	;
--	--	---	---

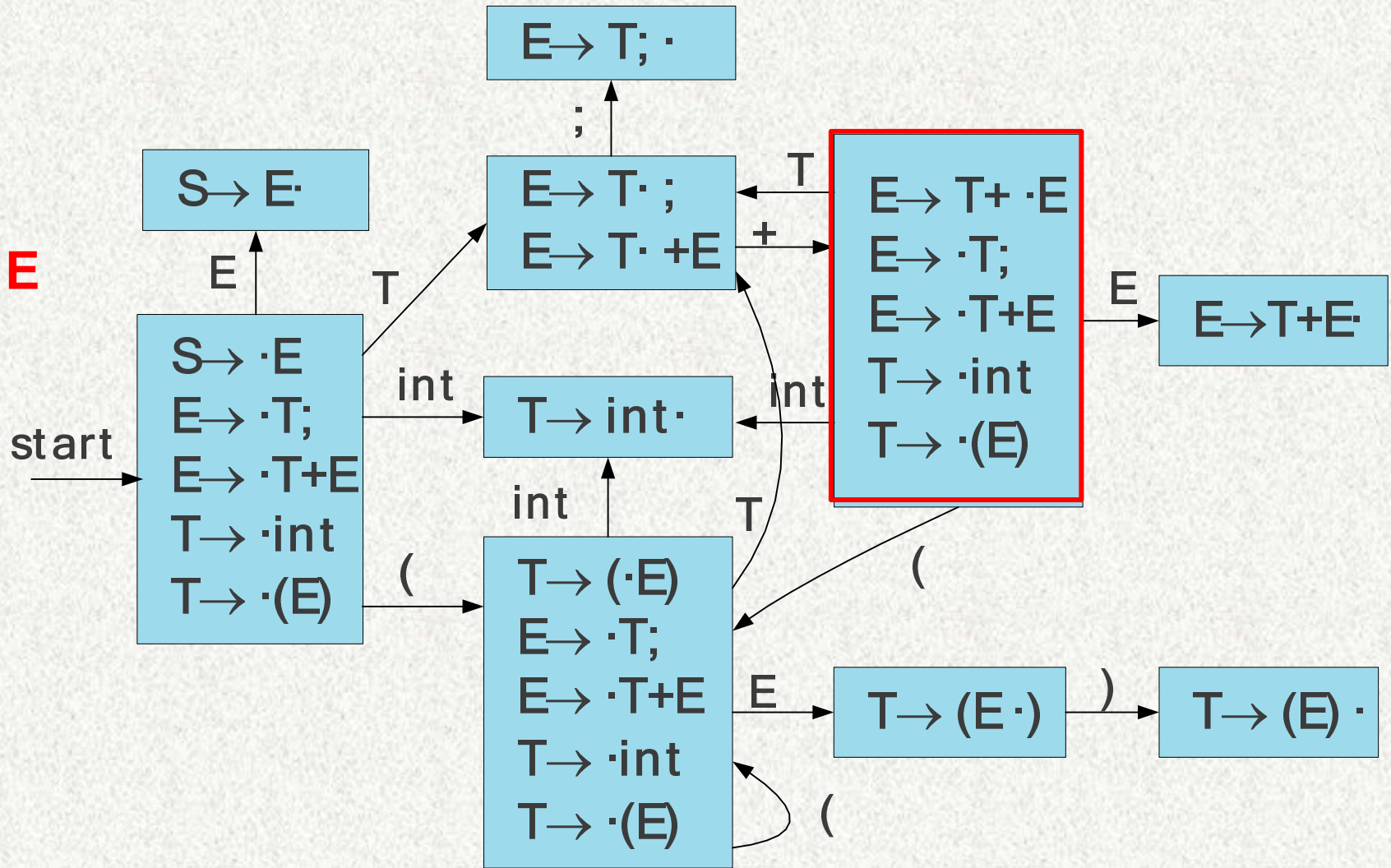
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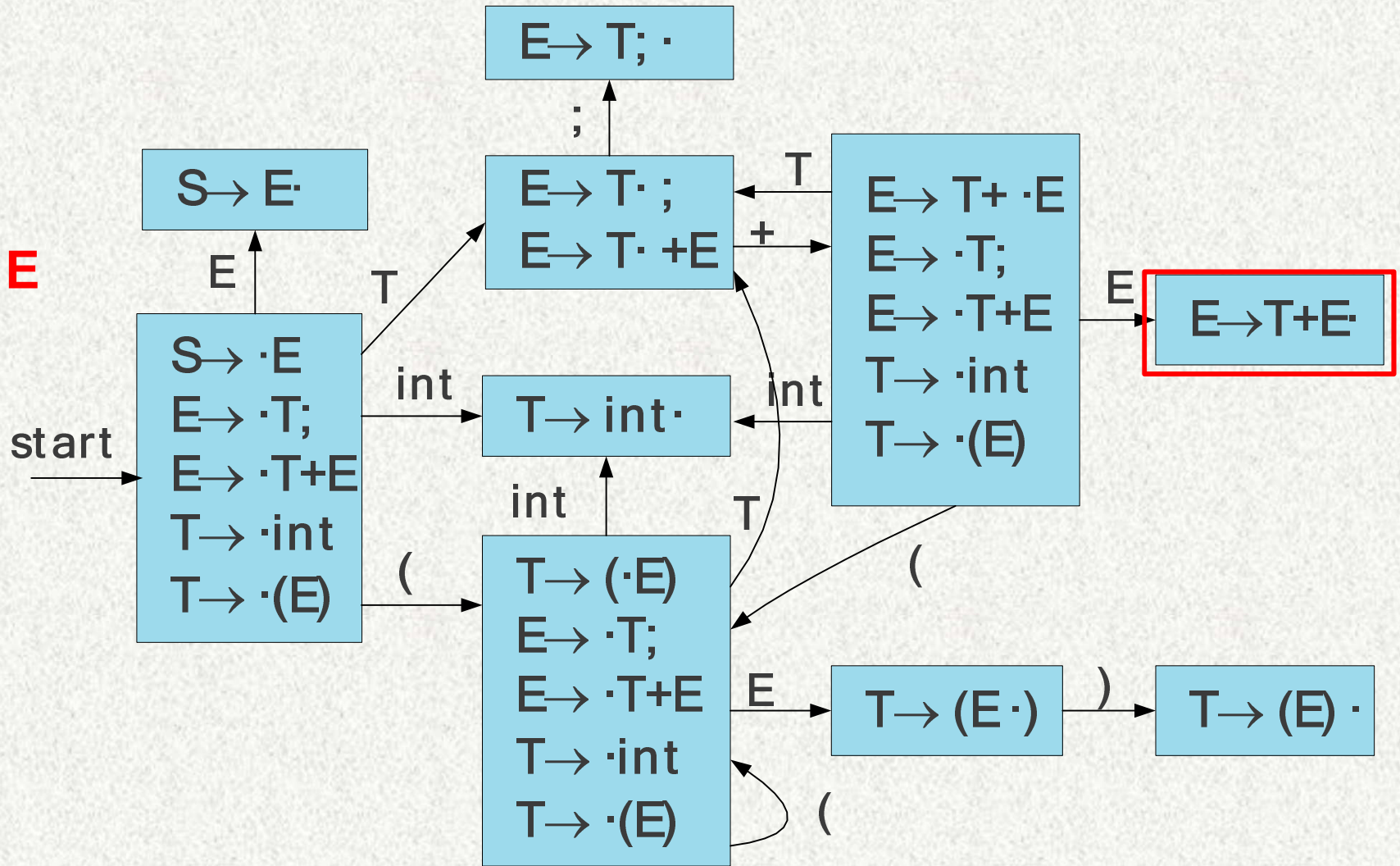
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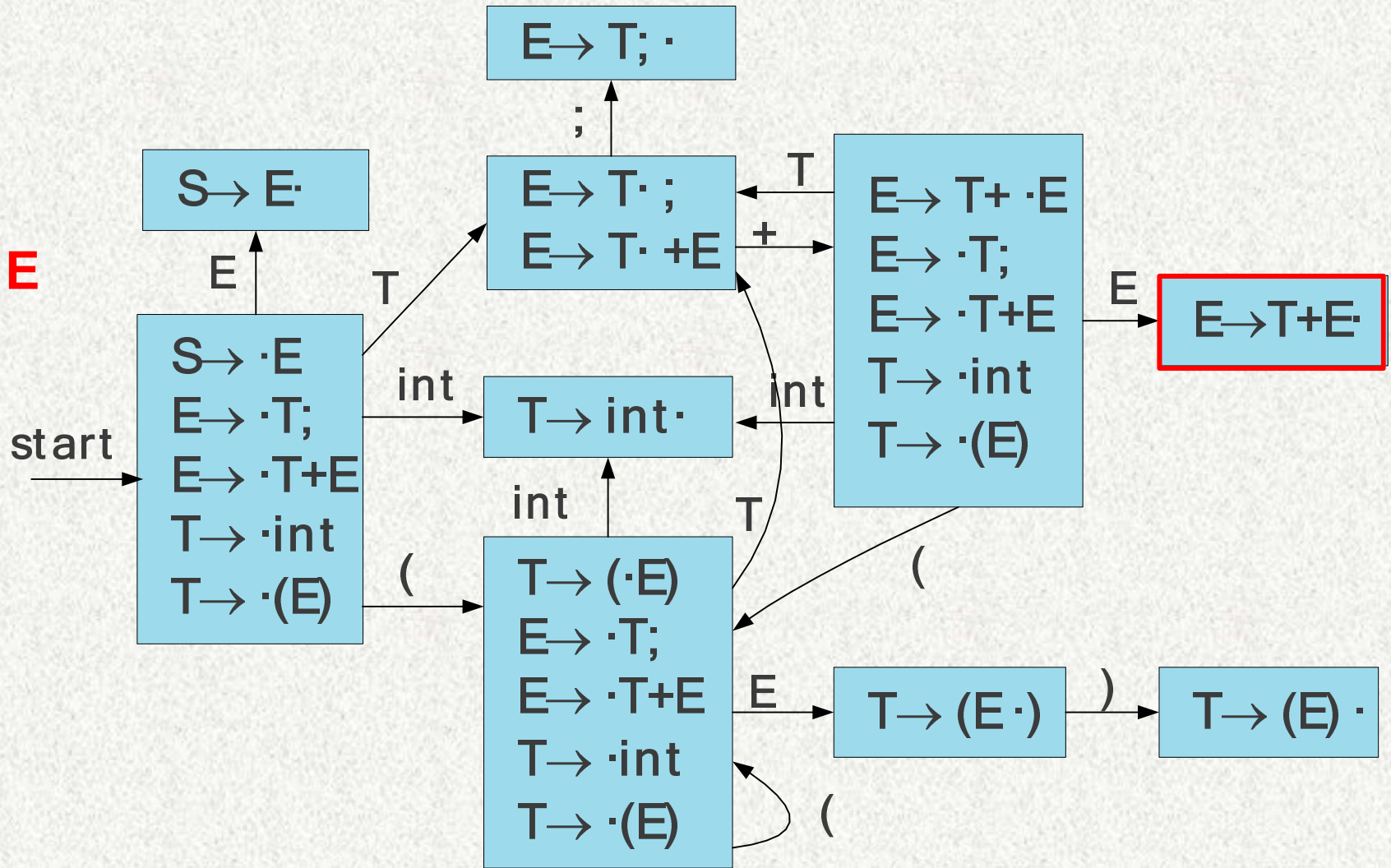
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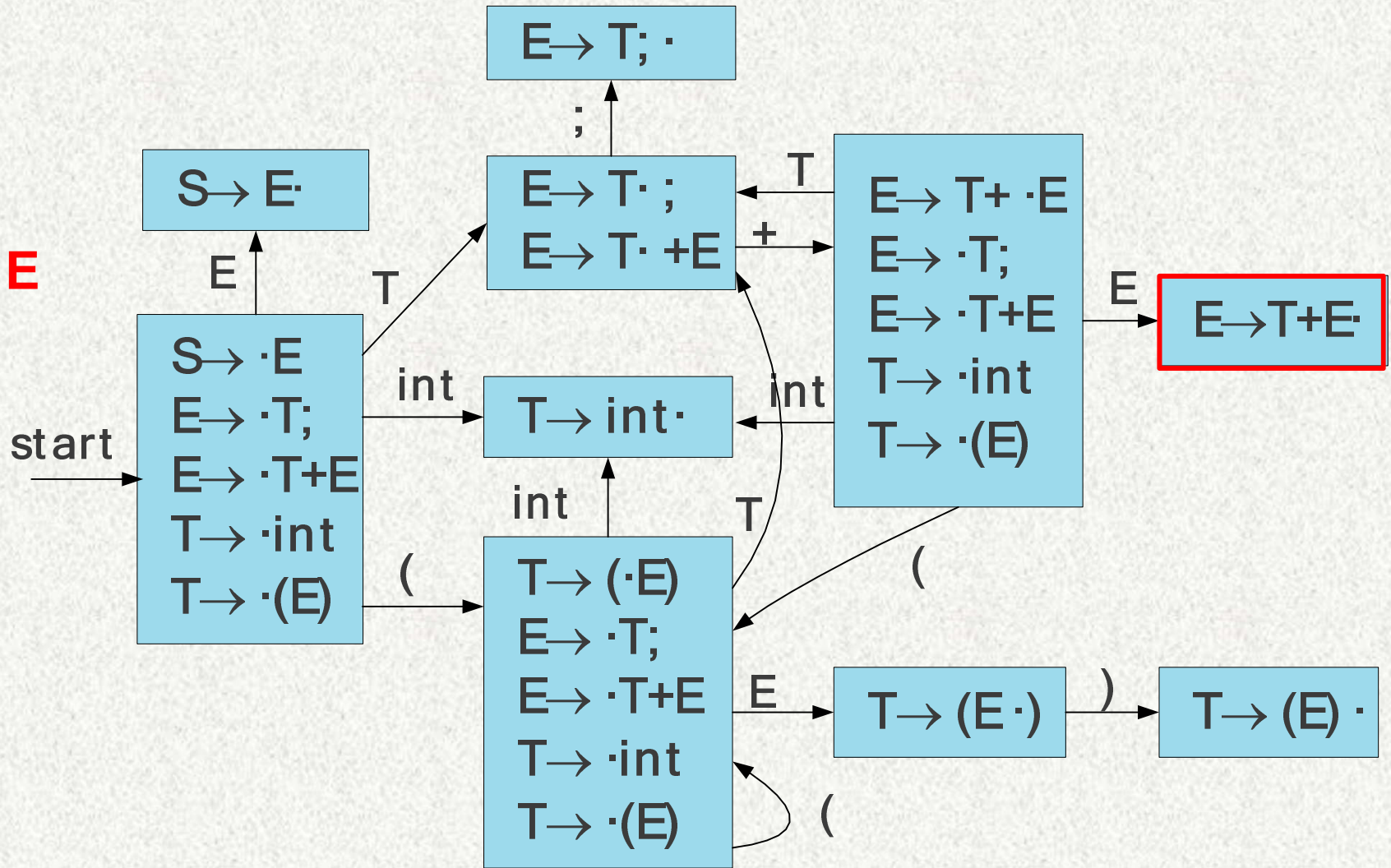


T	+	(
---	---	---	--

)	;
--	--	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

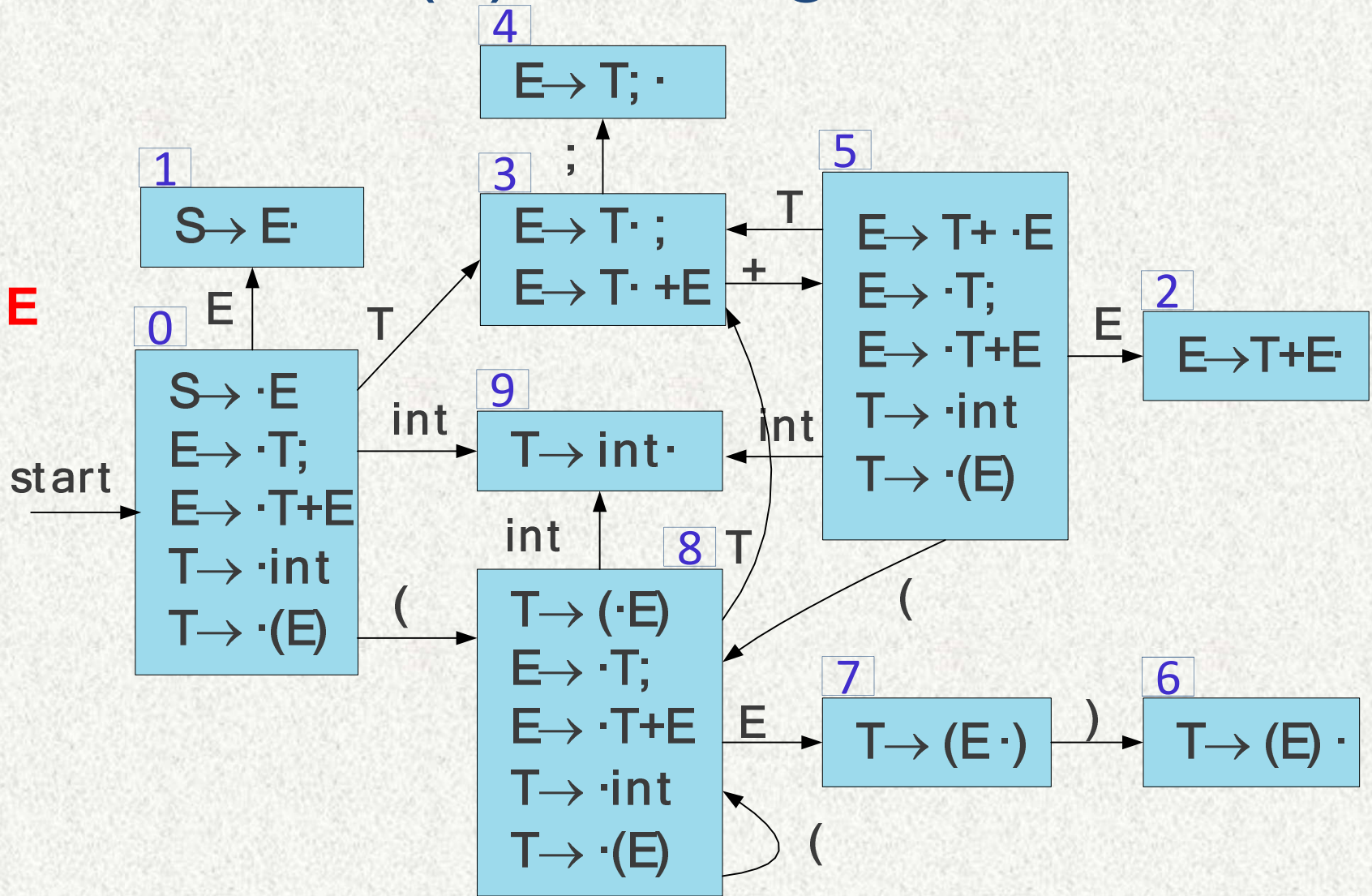


An optimization

- Rather than restart the automaton on each reduction, remember what state we were in for each symbol.
- When applying a reduction, restart the automaton from the last known good state.

LR(0) Parsing

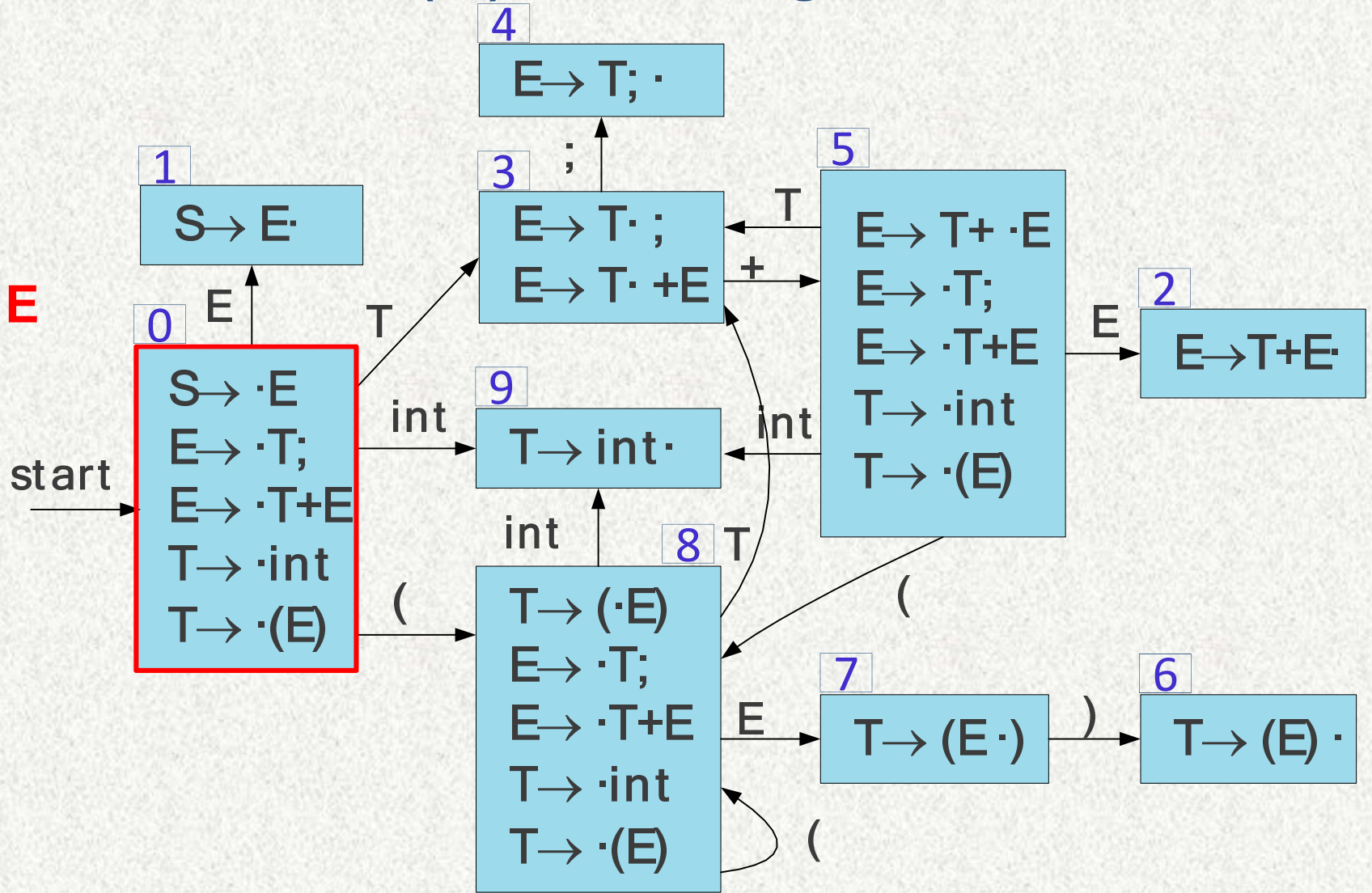
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



int	+	(int	+	int	;)	;
-----	---	---	-----	---	-----	---	---	---

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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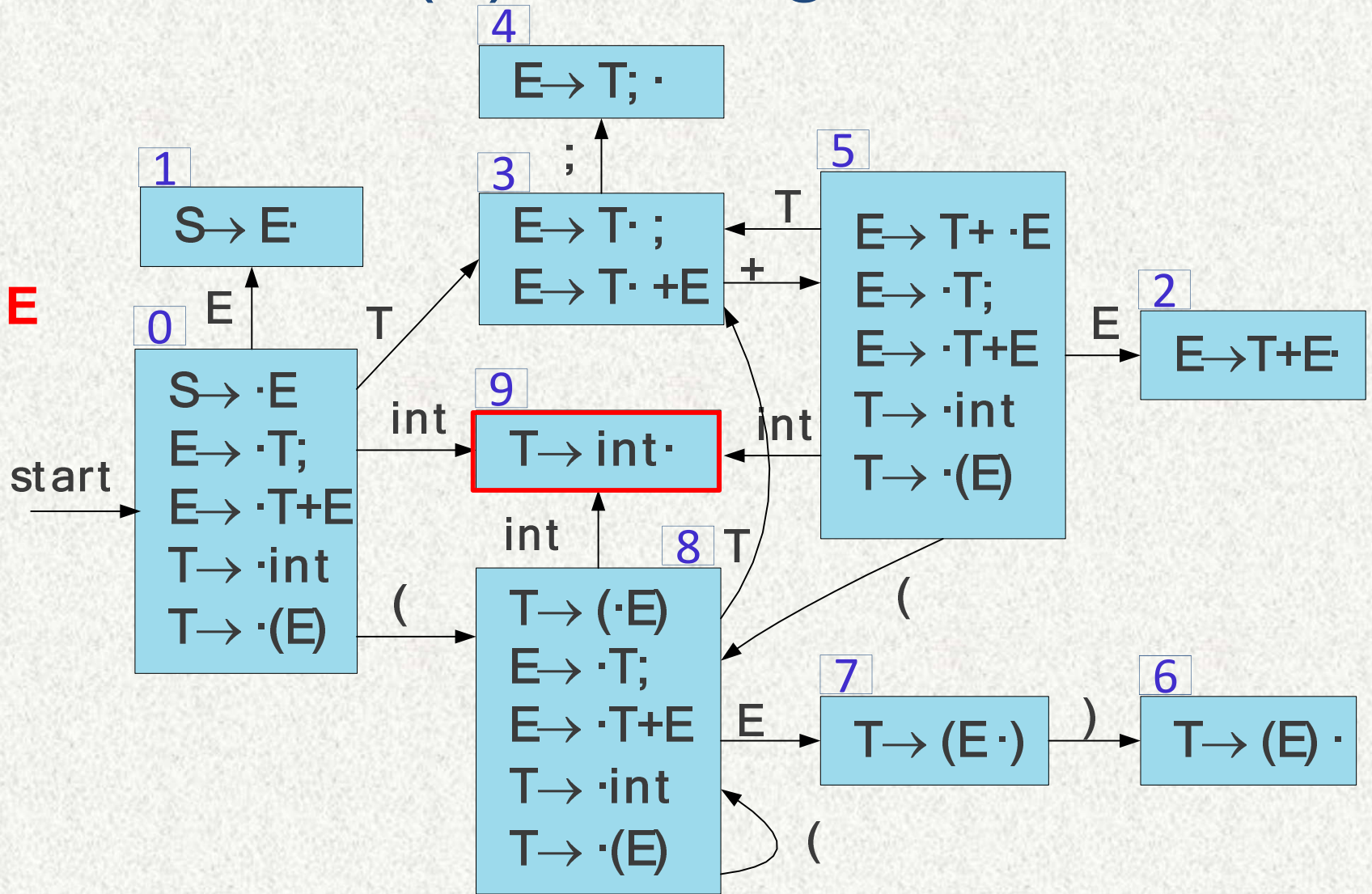


$\$$
 0

int	+	(int	+	int	;)	;	\$
-----	---	---	-----	---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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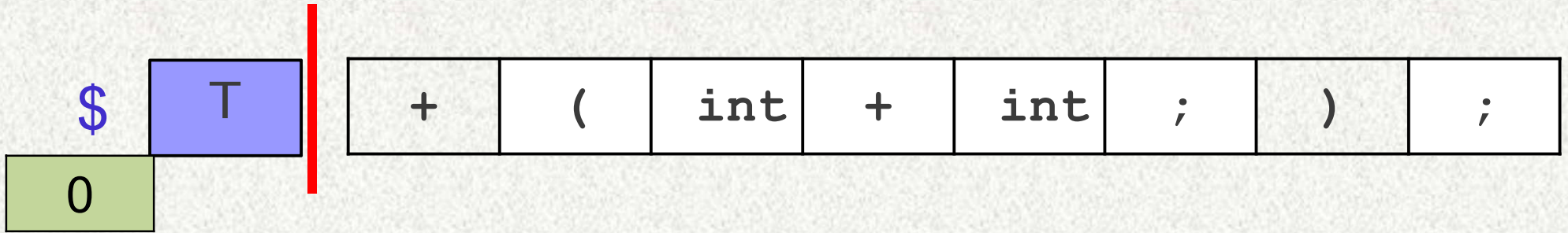
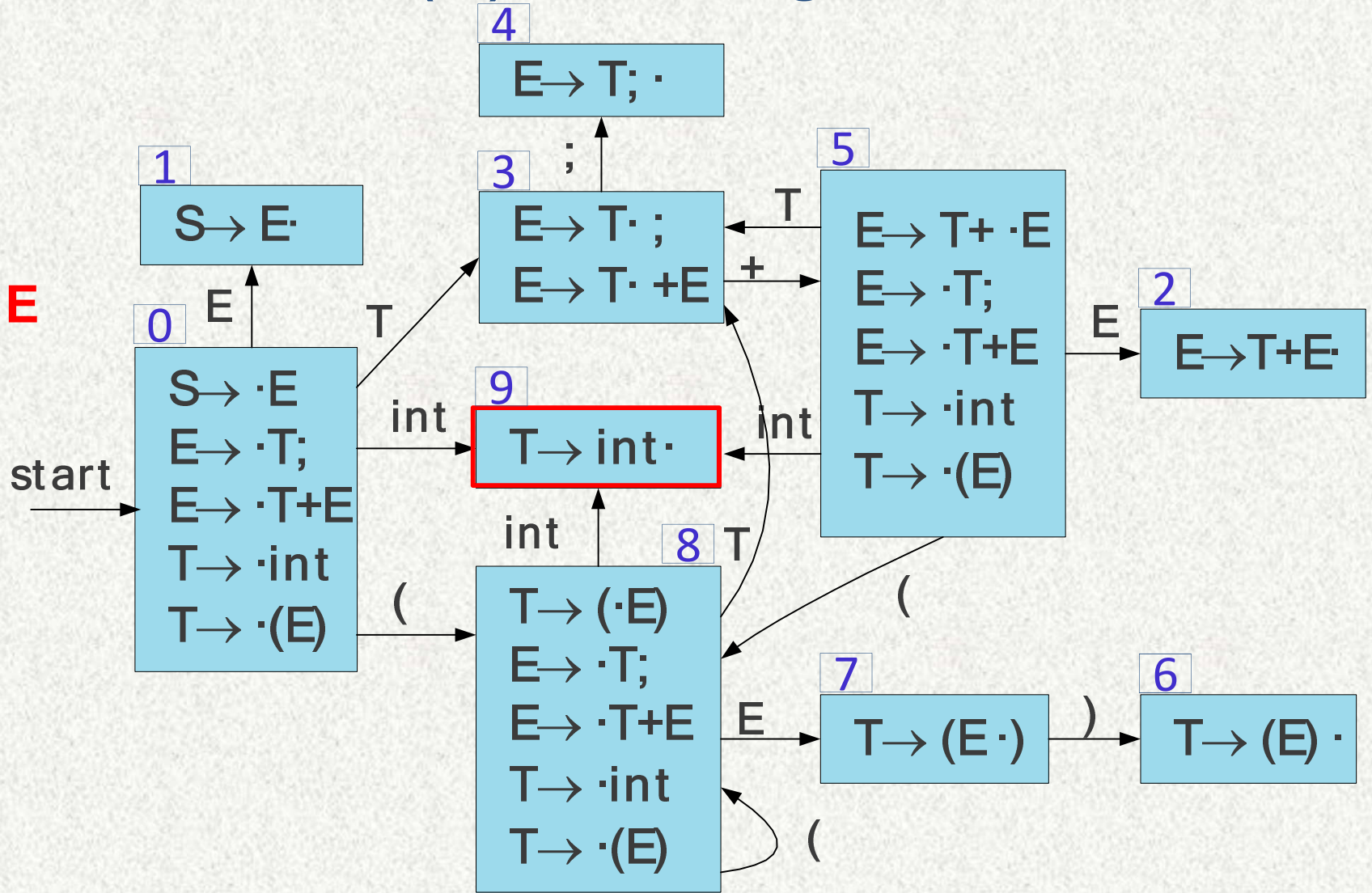
\$

0

	+	(int	+	int	;)	;	\$
--	---	---	-----	---	-----	---	---	---	----

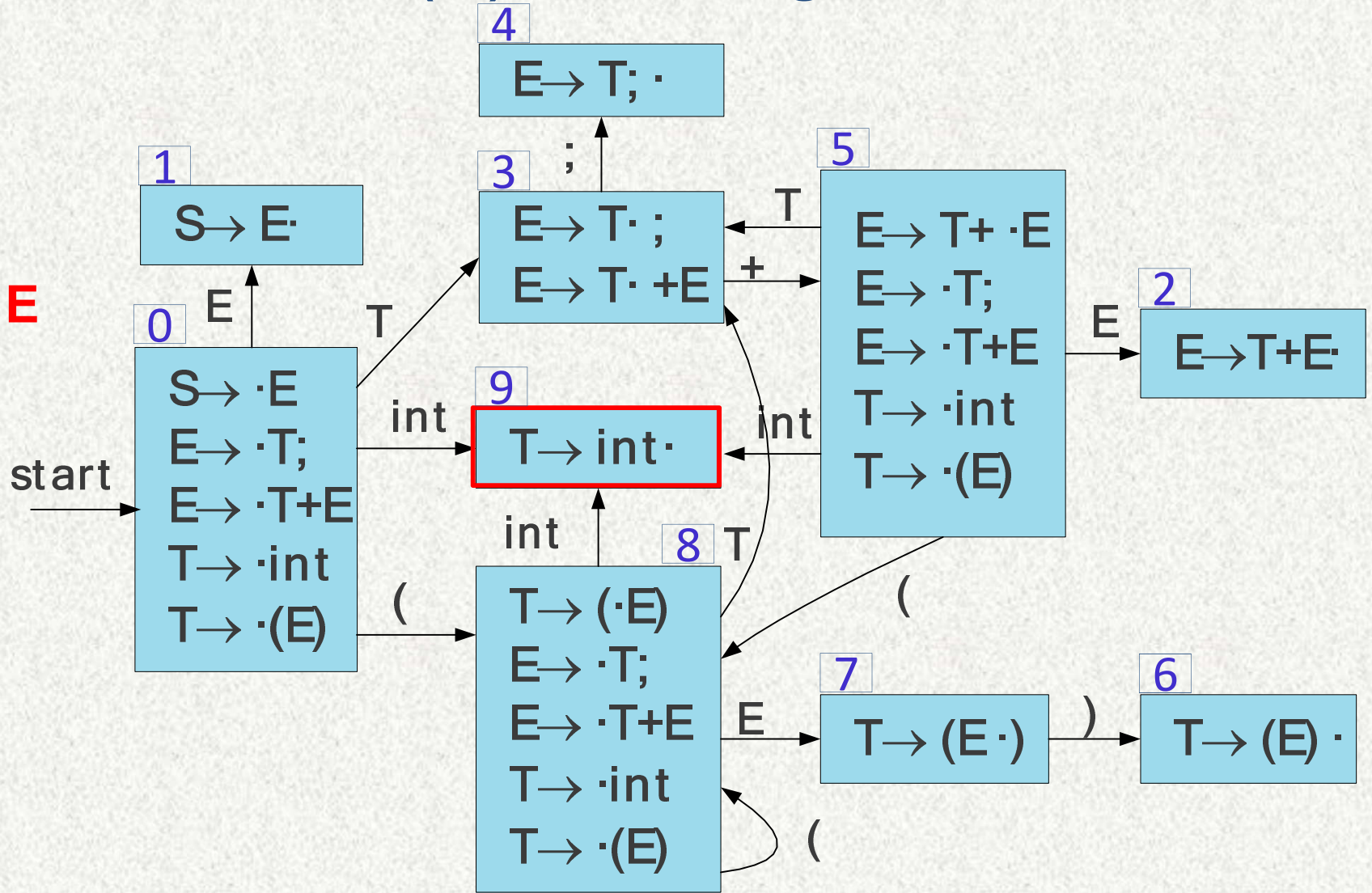
LR(0) Parsing

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LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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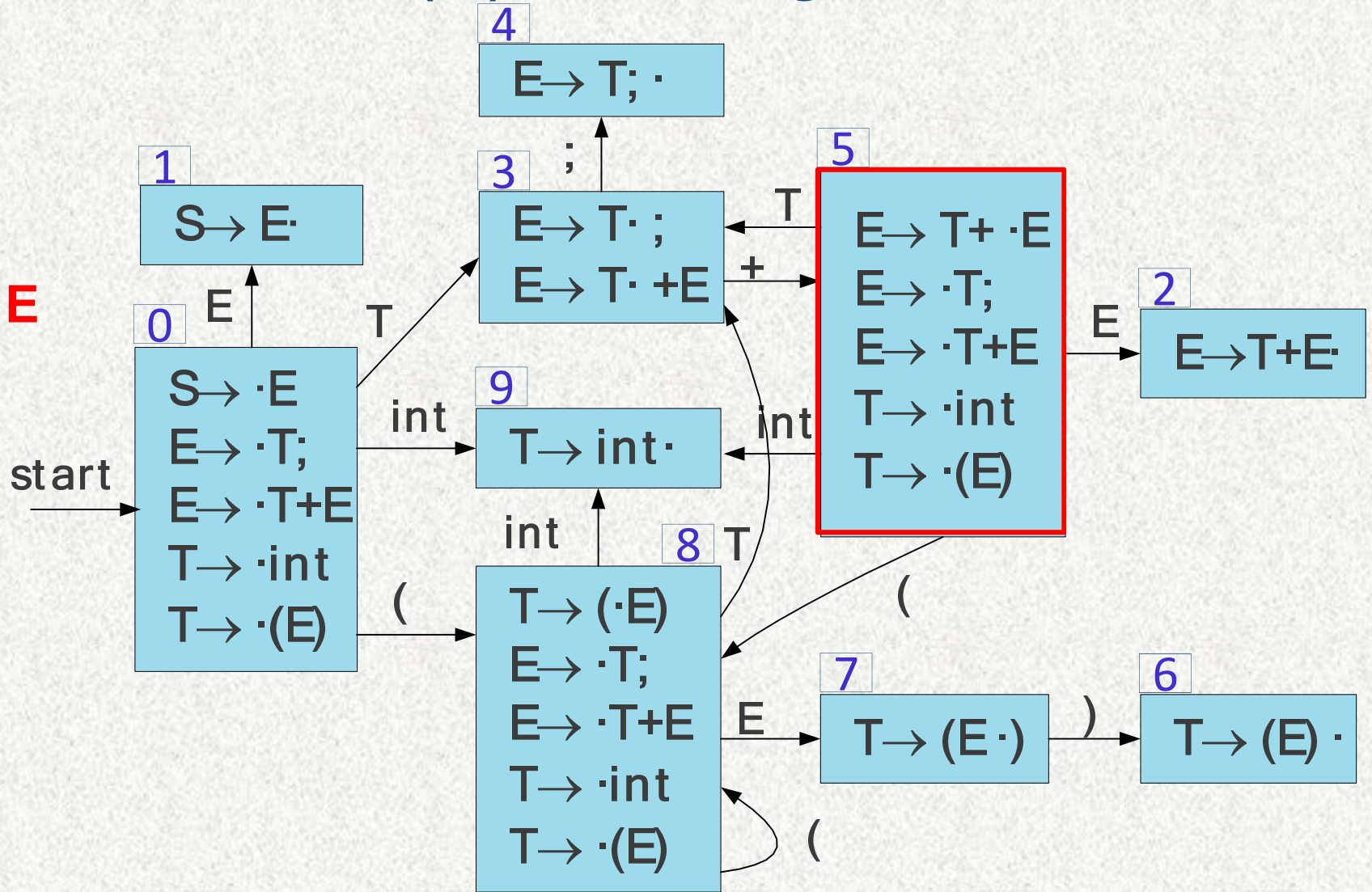


\$	T
0	3

+ (int + int ;) ; \$

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

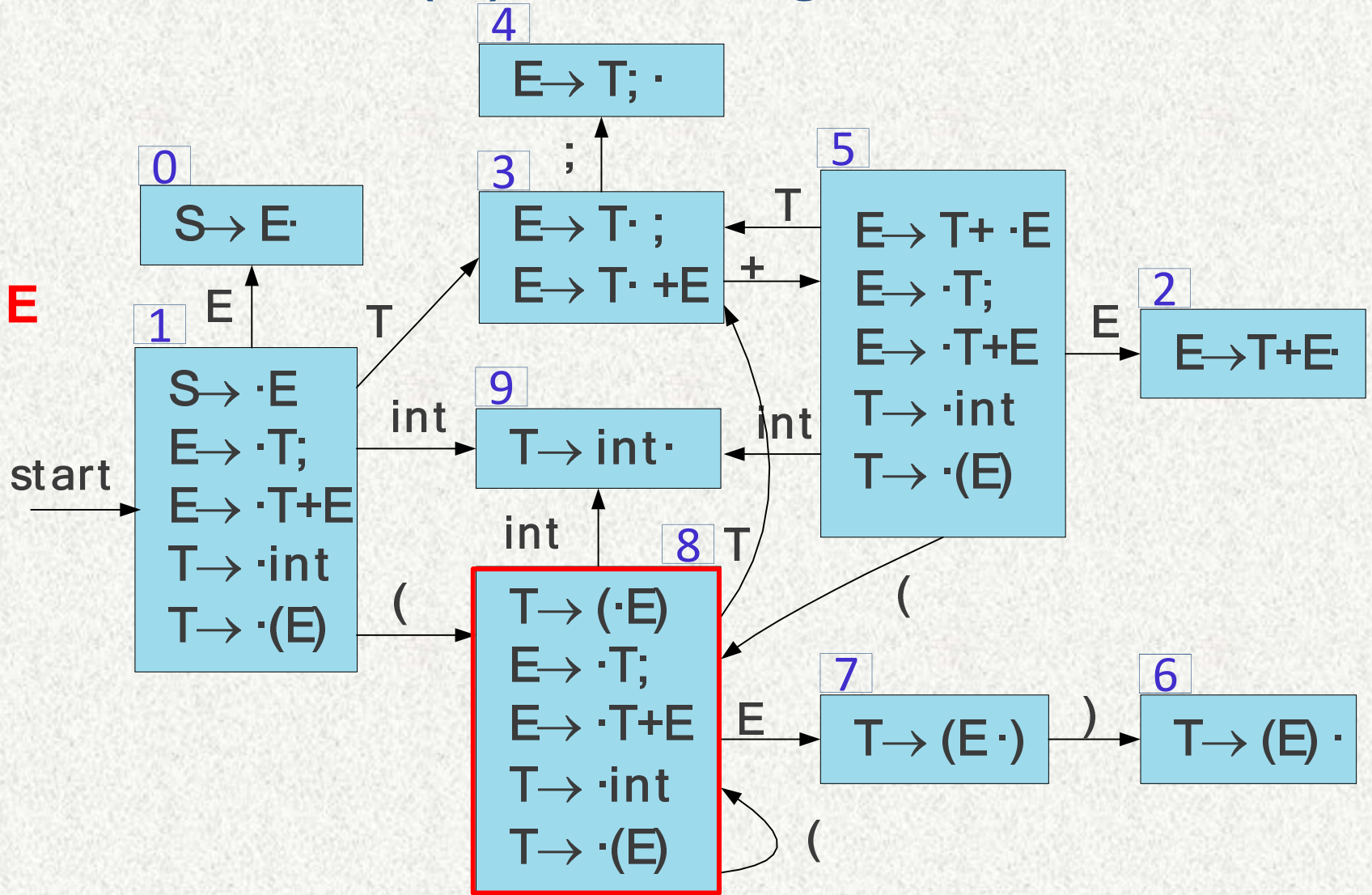


\$	T	+
0	3	5

(int	+	int	;)	;	\$
---	-----	---	-----	---	---	---	----

LR(0) Parsing

S → **E**
E → **T**;
E → **T + E**
T → **int**
T → **(E)**

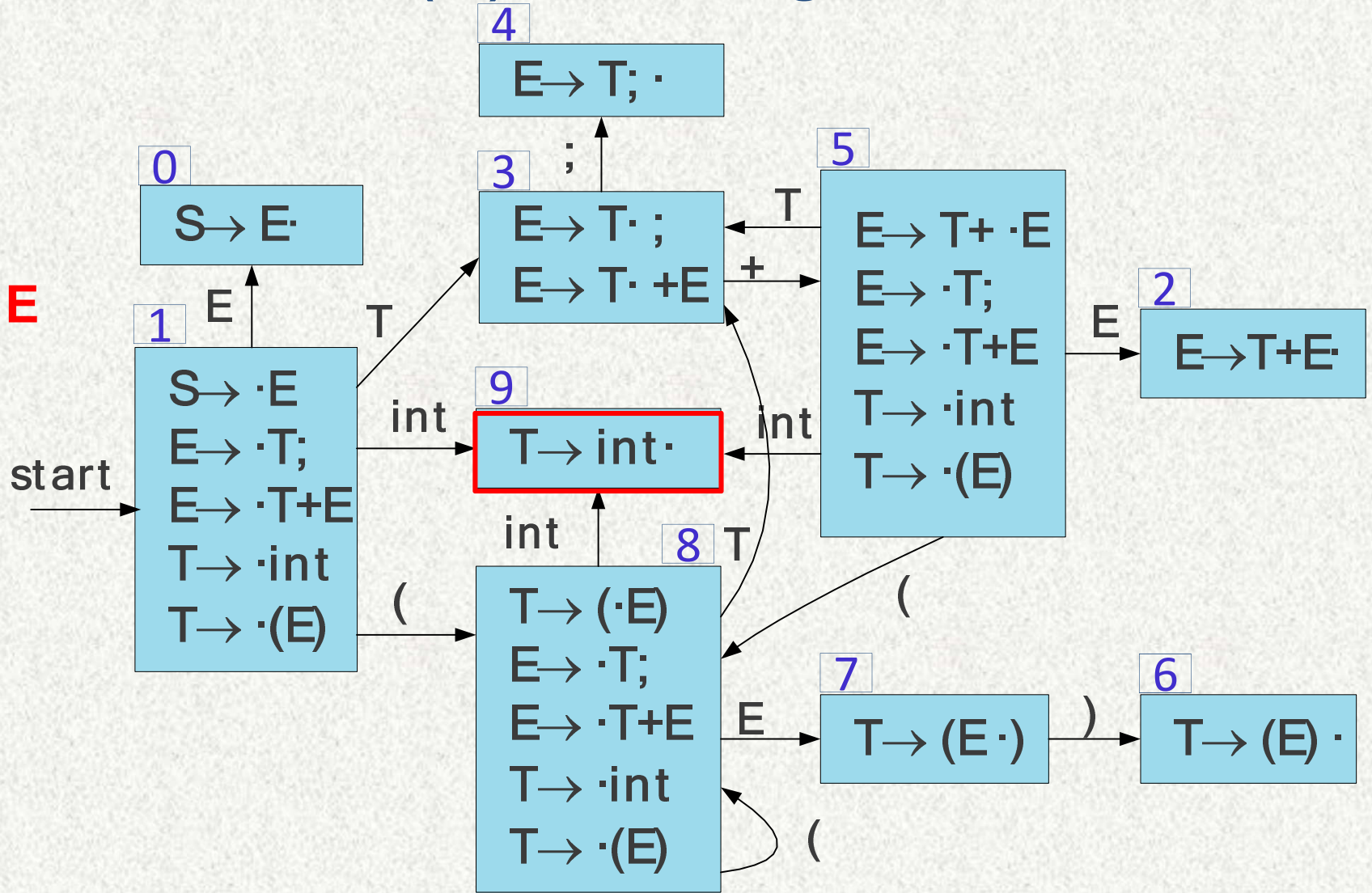


\$	T	+	(
0	3	5	8

int	+	int	;)	;	\$
-----	---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

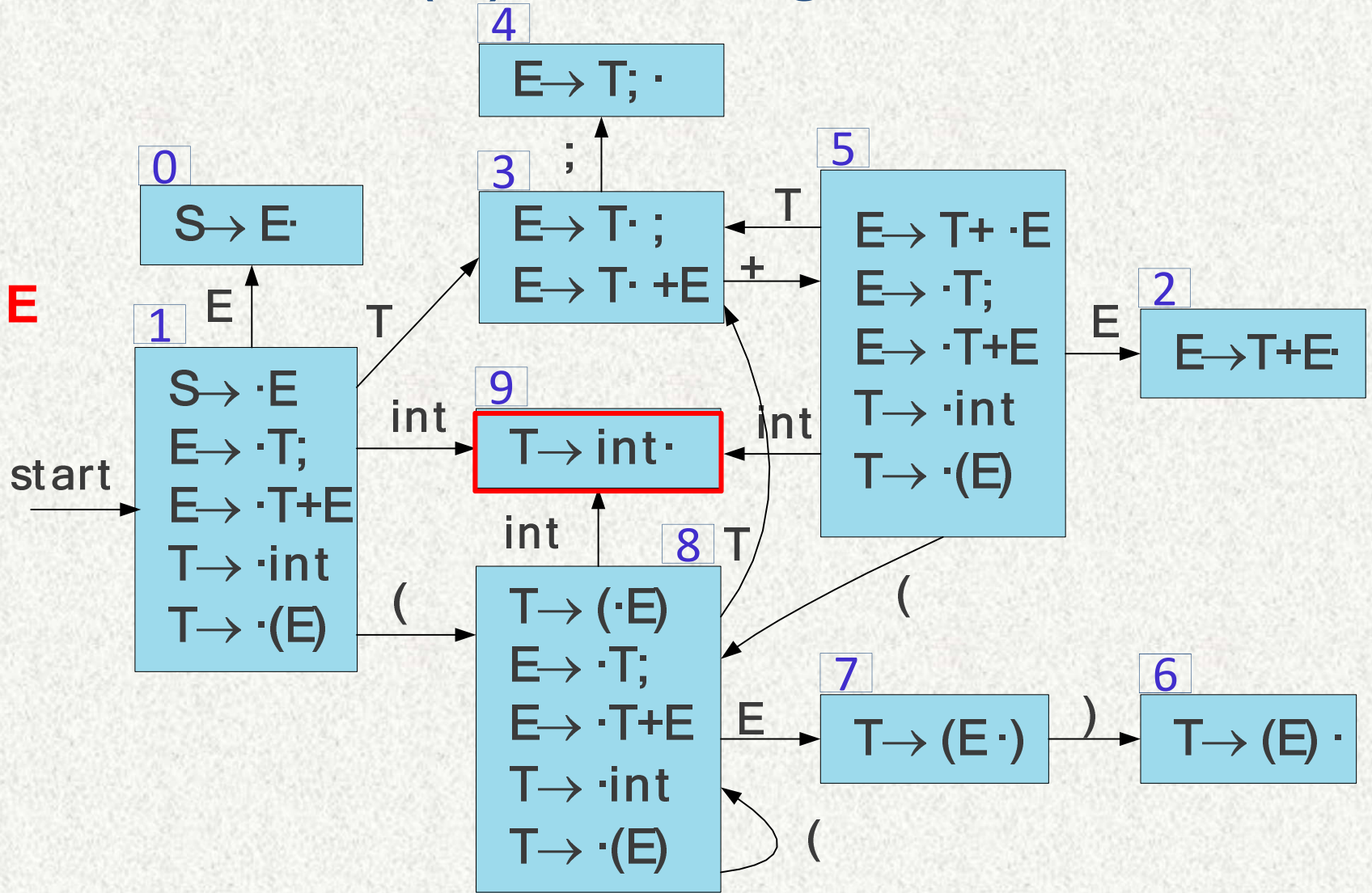


\$	T	+	(int
0	3	5	8	9

	+	int	;)	;	\$
--	---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

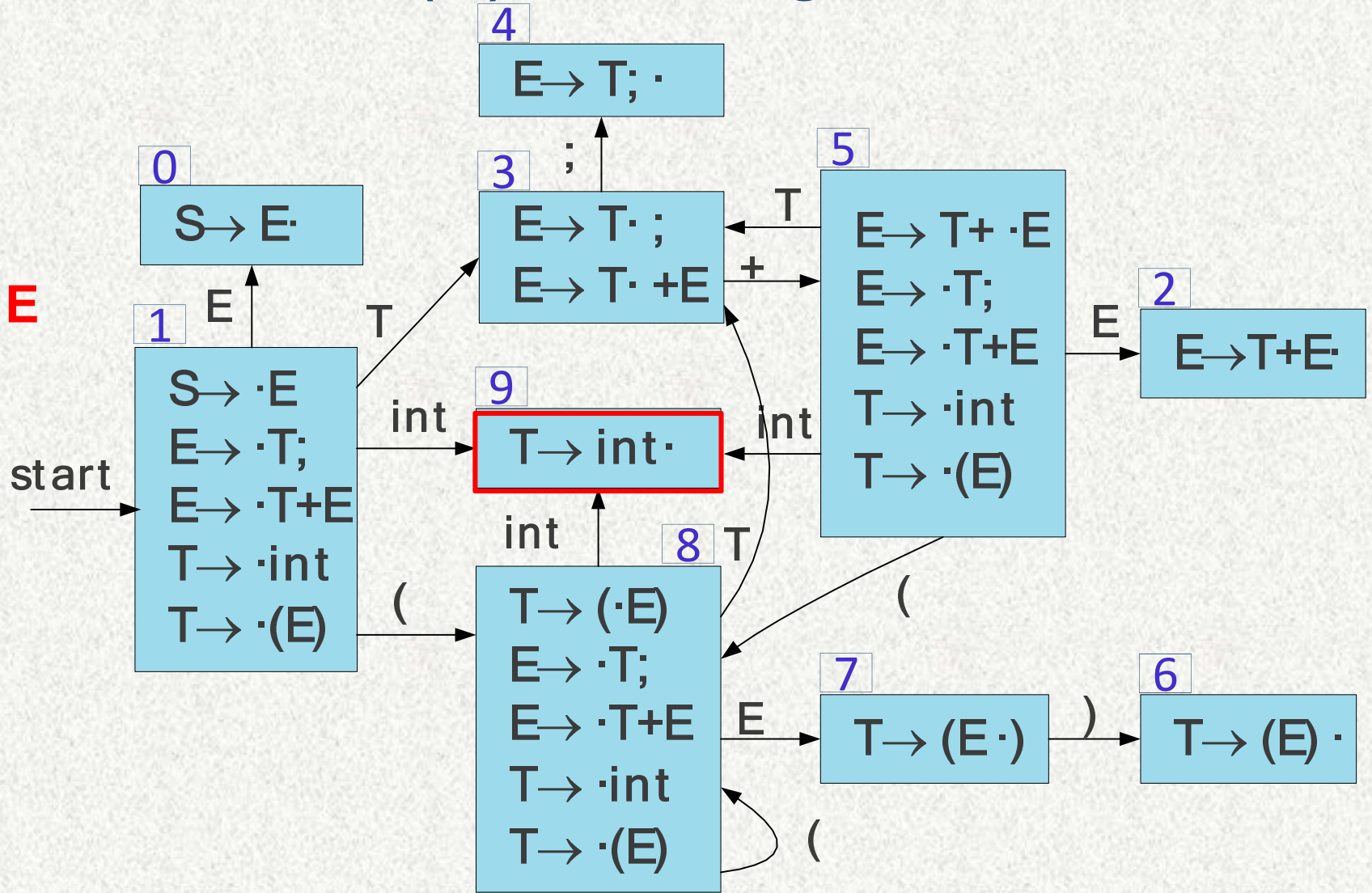


\$	T	+	(
0	3	5	8	

	+	int	;)	;	\$
--	---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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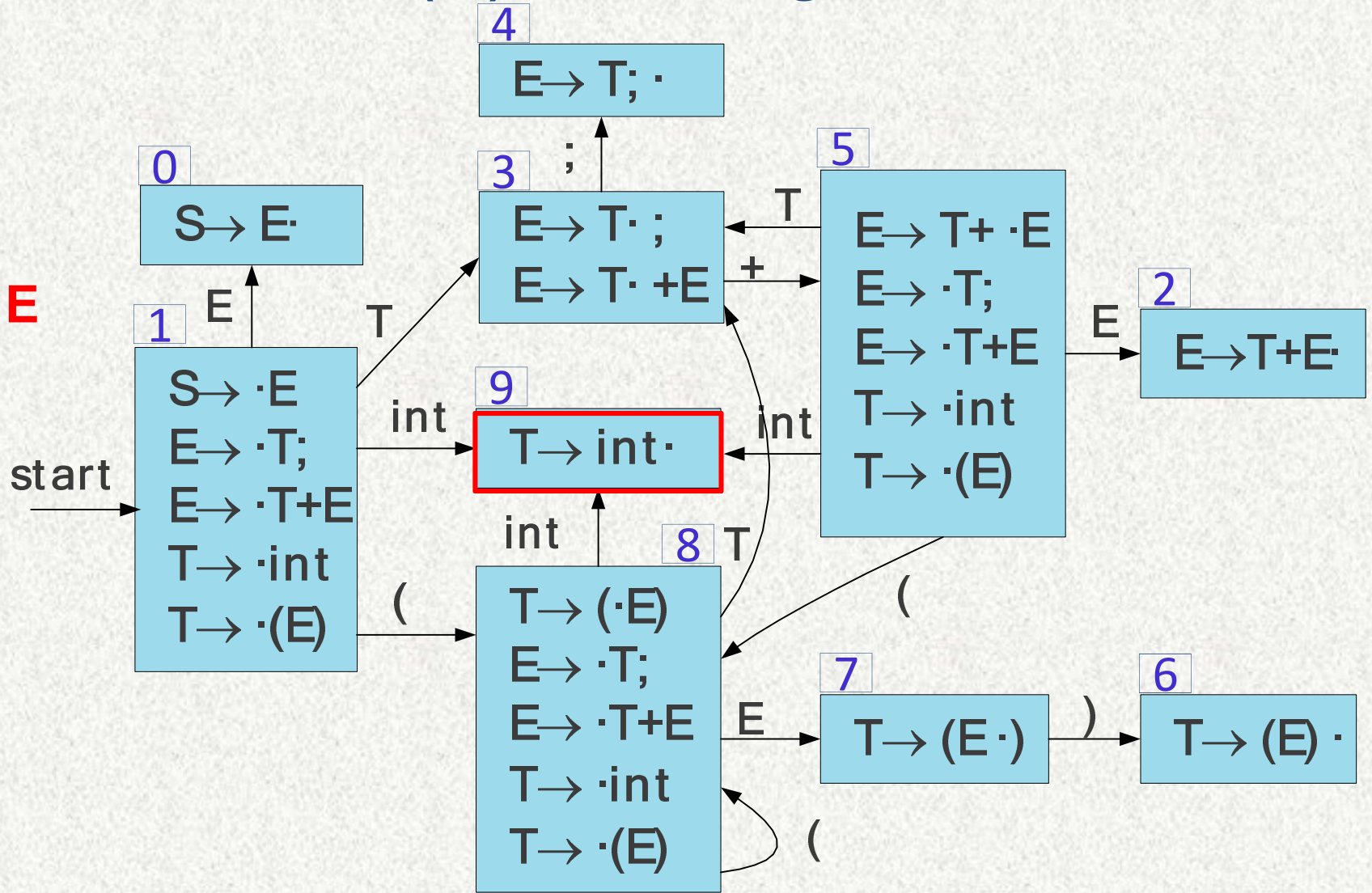


\$	T	+	(T
0	3	5	8	

	+	int	;)	;	\$
--	---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
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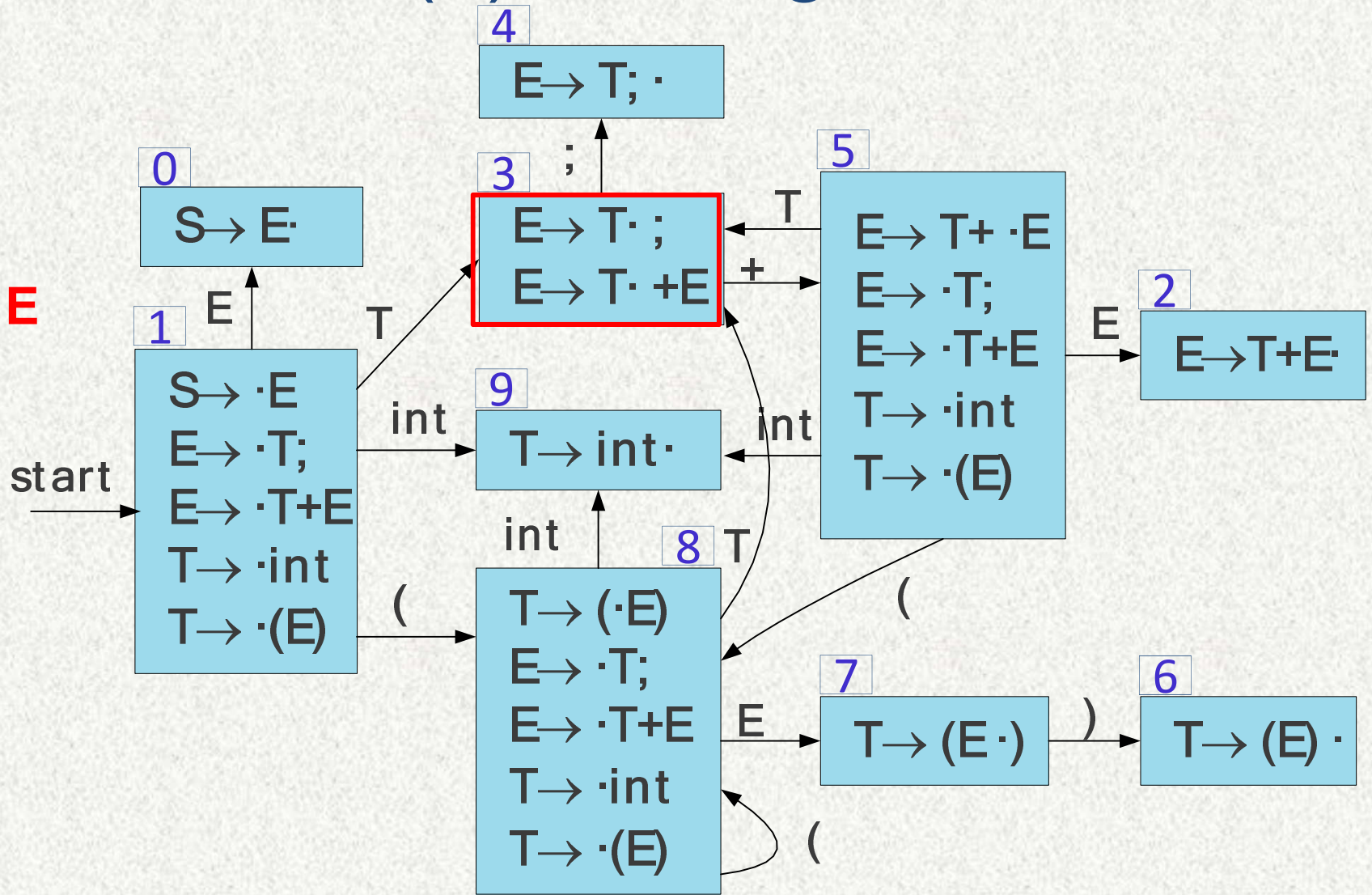


\$	T	+	(T
0	3	5	8	3

+	int	;)	;	\$
---	-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

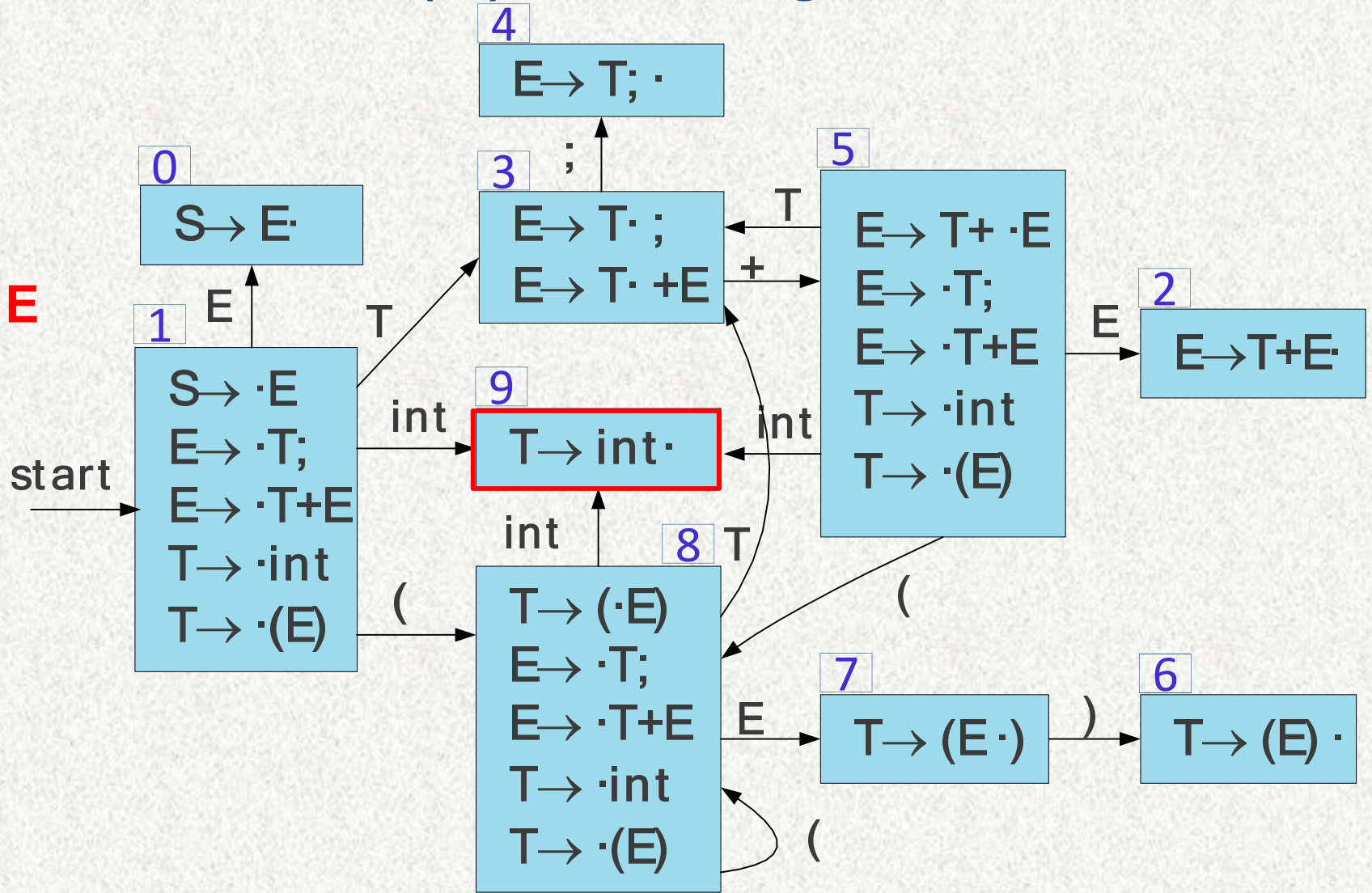


\$	T	+	(T	+
0	3	5	8	3	5

int	;)	;	\$
-----	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

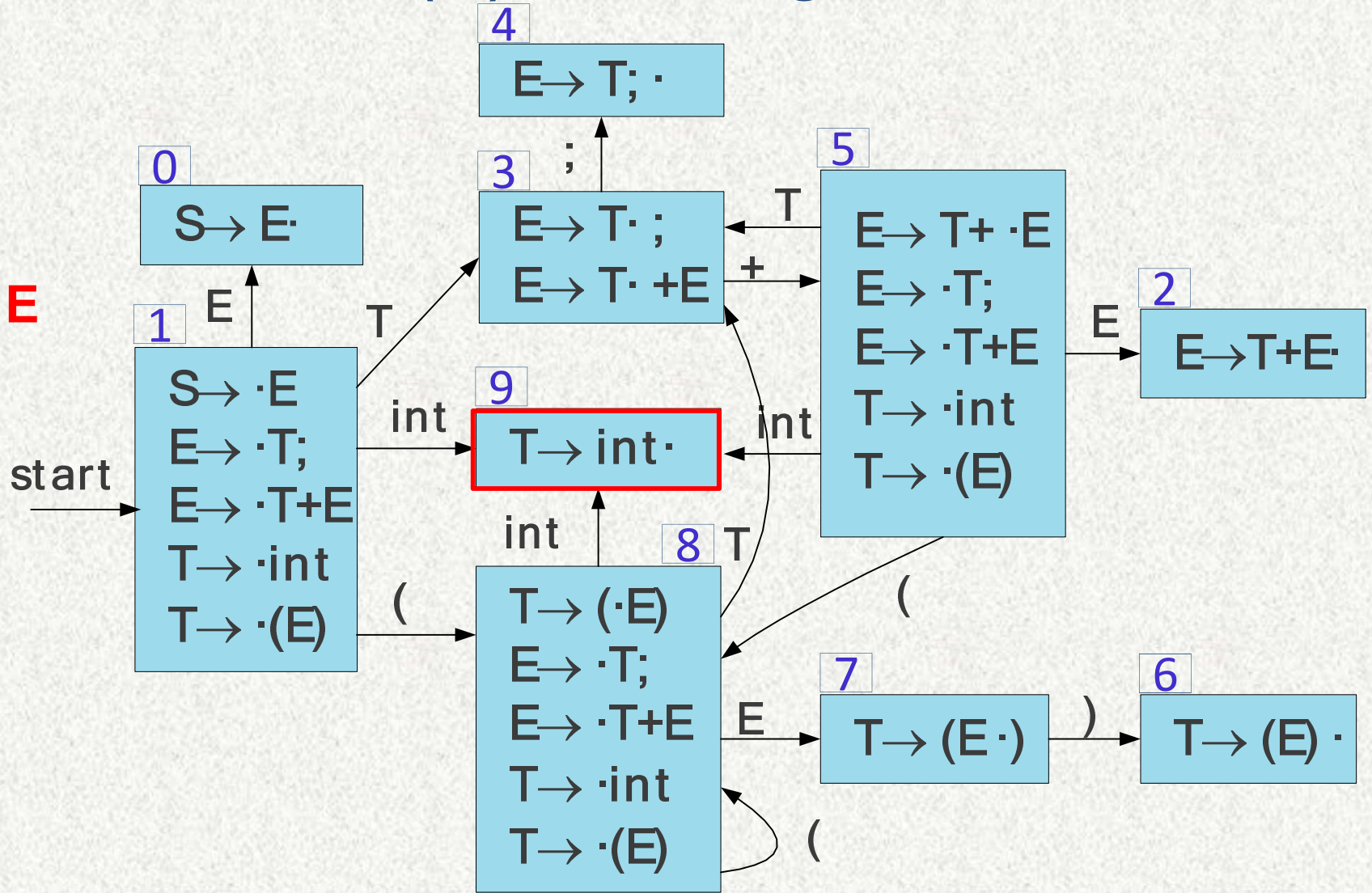


\$	T	+	(T	+	int
0	3	5	8	3	5	9

	;)	;	\$
--	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

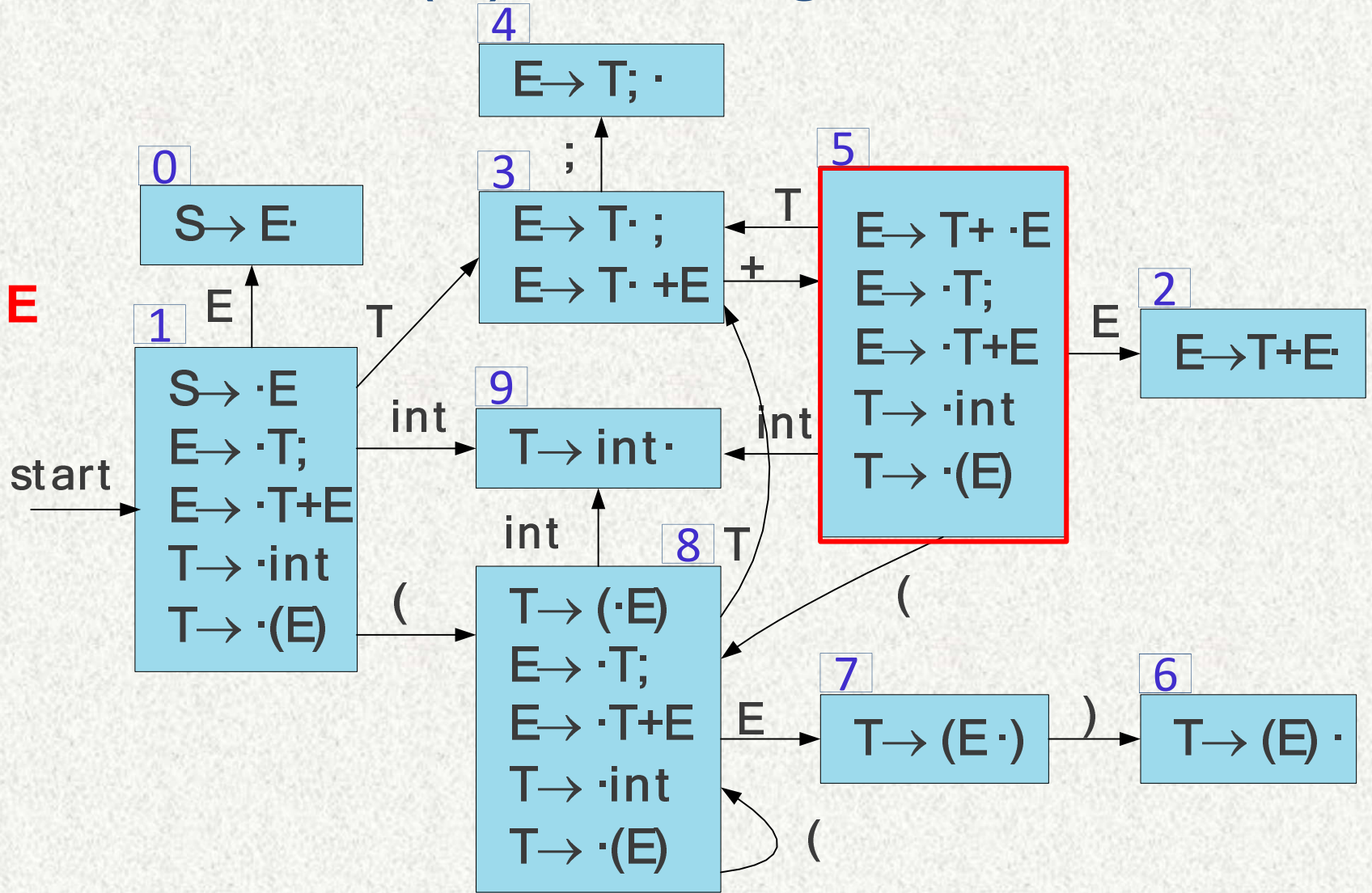


\$	T	+	(T	+
0	3	5	8	3	5

	;)	;	\$
--	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

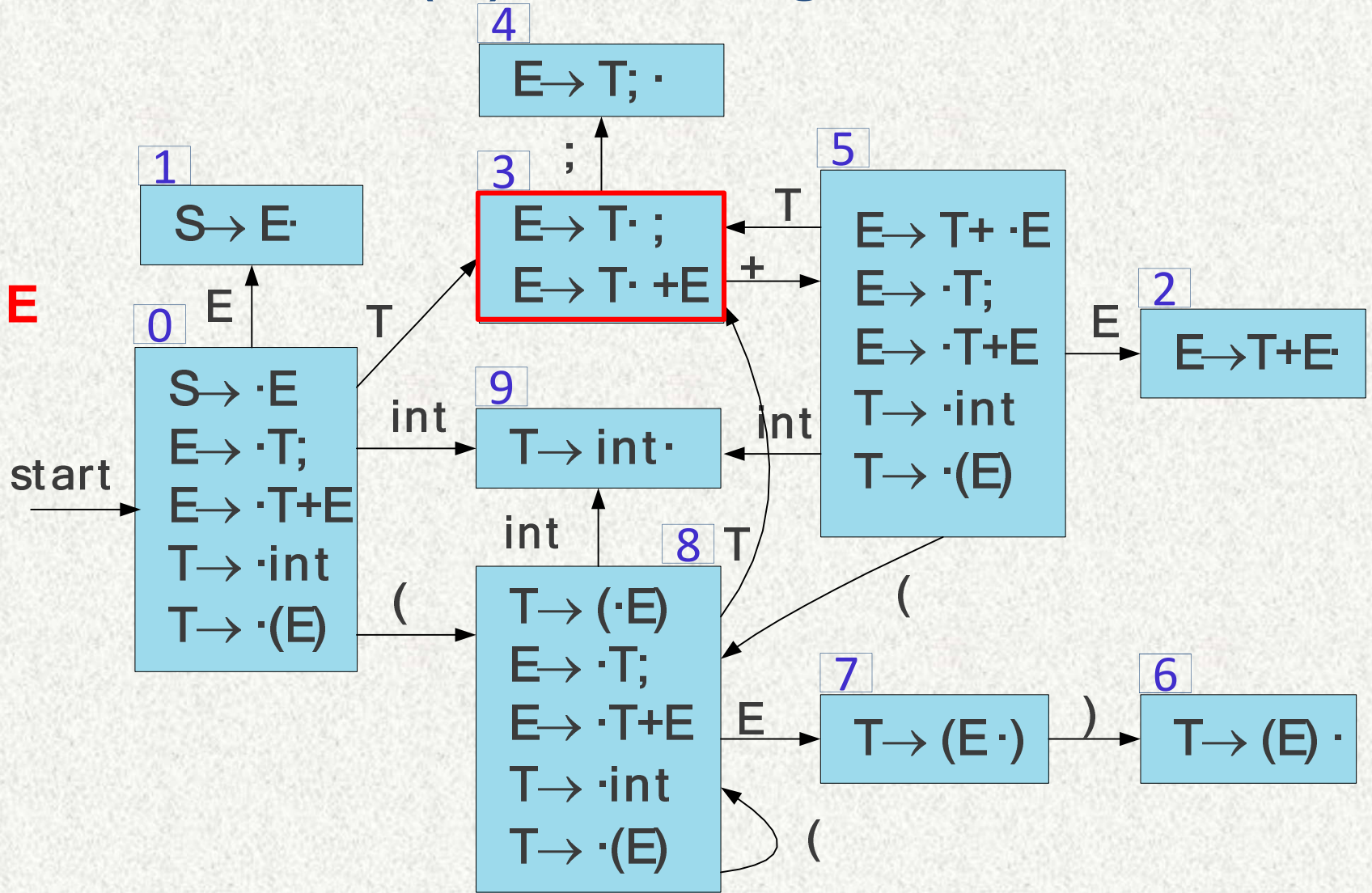


\$	T	+	(T	+	T
0	3	5	8	3	5	

	;)	;	\$
--	---	---	---	----

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

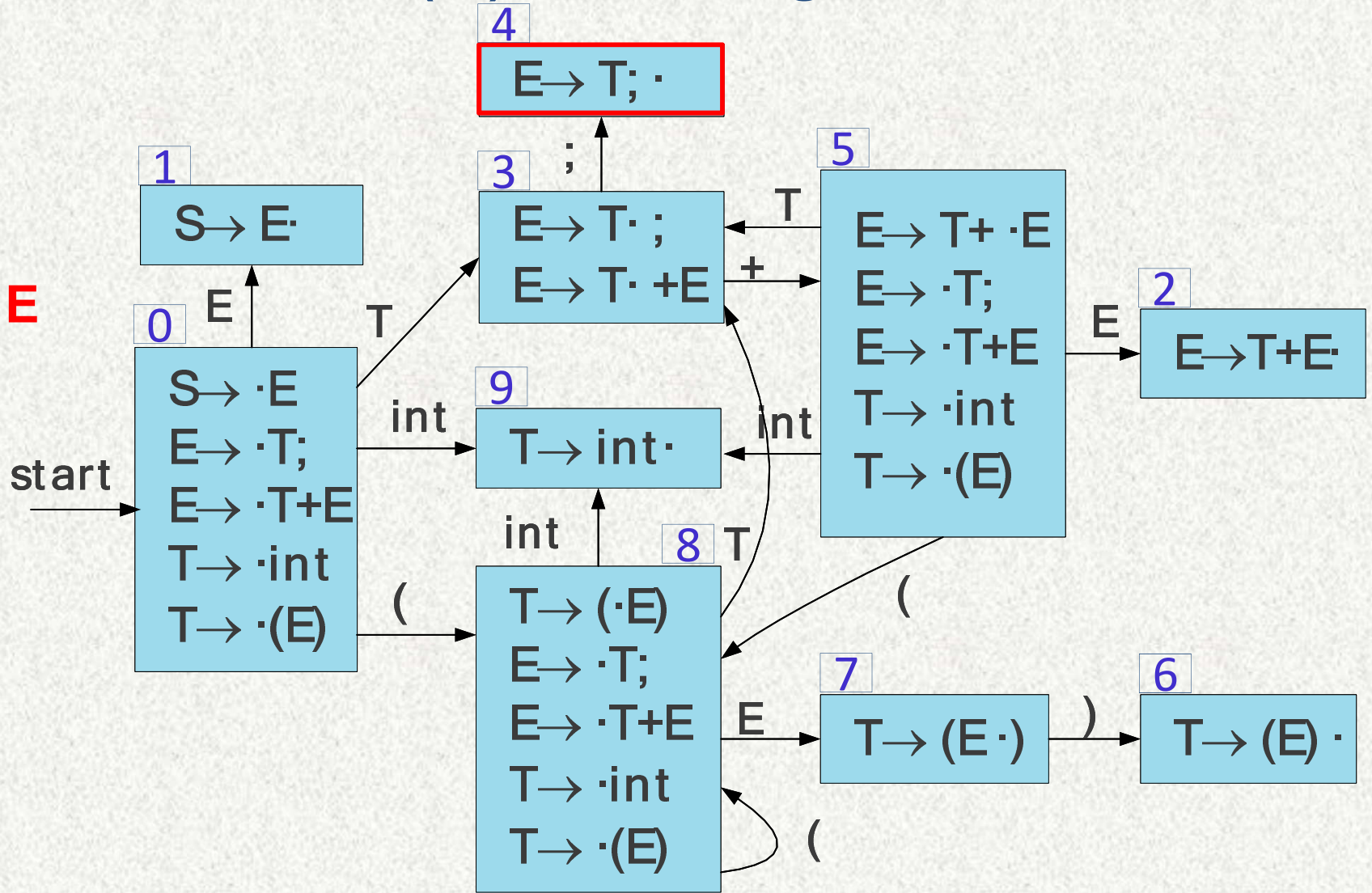


\$	T	+	(T	+	T
0	3	5	8	3	5	3

	;)	;	\$
--	---	---	---	----

LR(0) Parsing

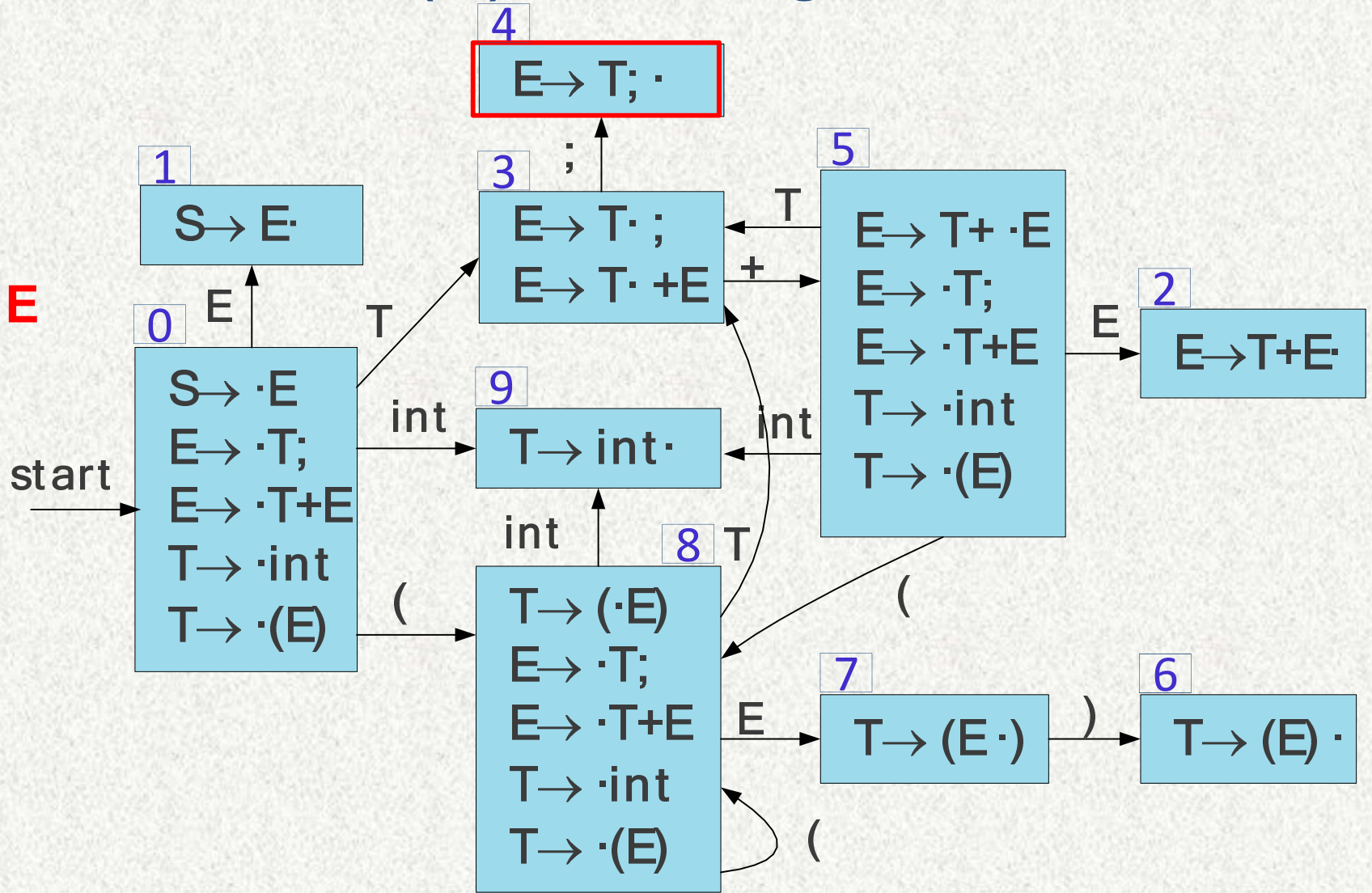
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



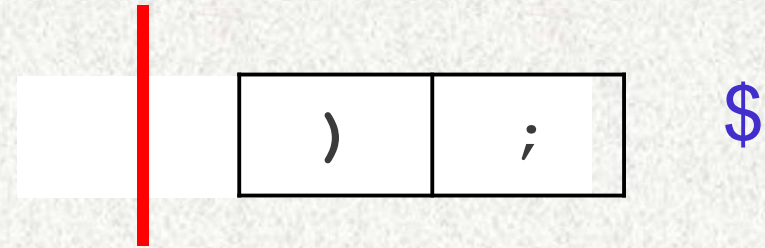
\$	T	+	(T	+	T	;)	;	\$
0	3	5	8	3	5	3	4				

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

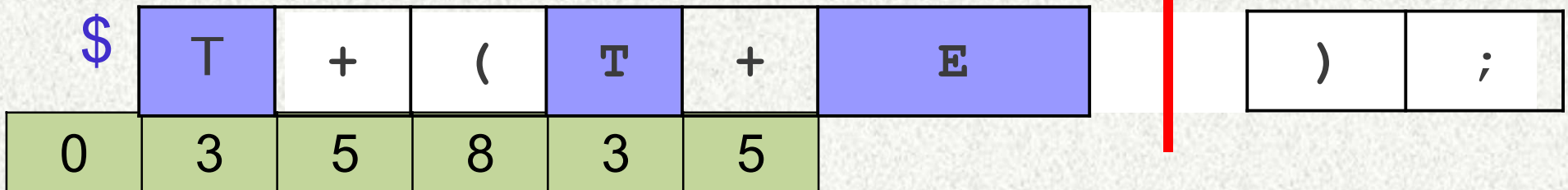
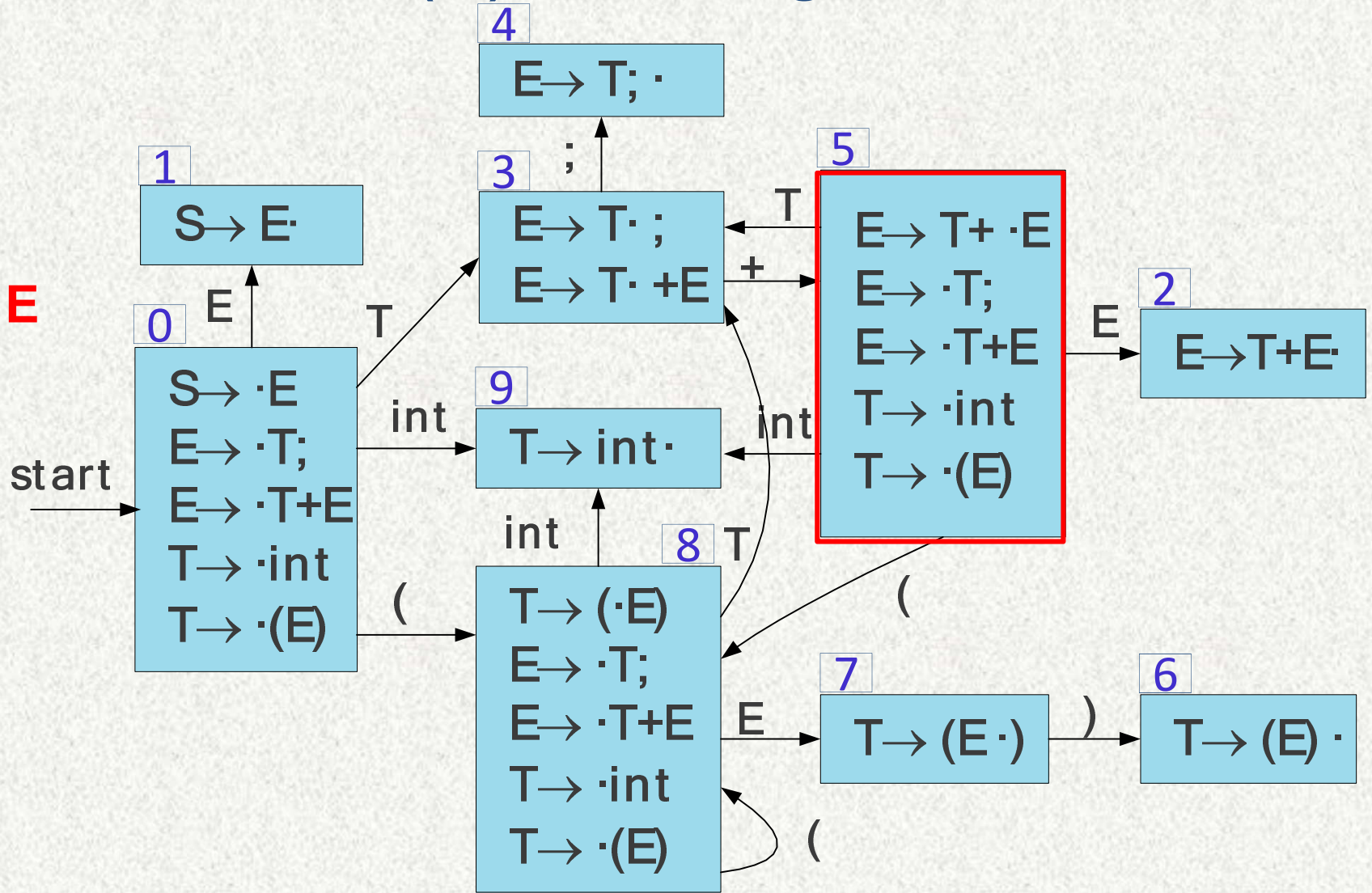


\$	T	+	(T	+
0	3	5	8	3	5



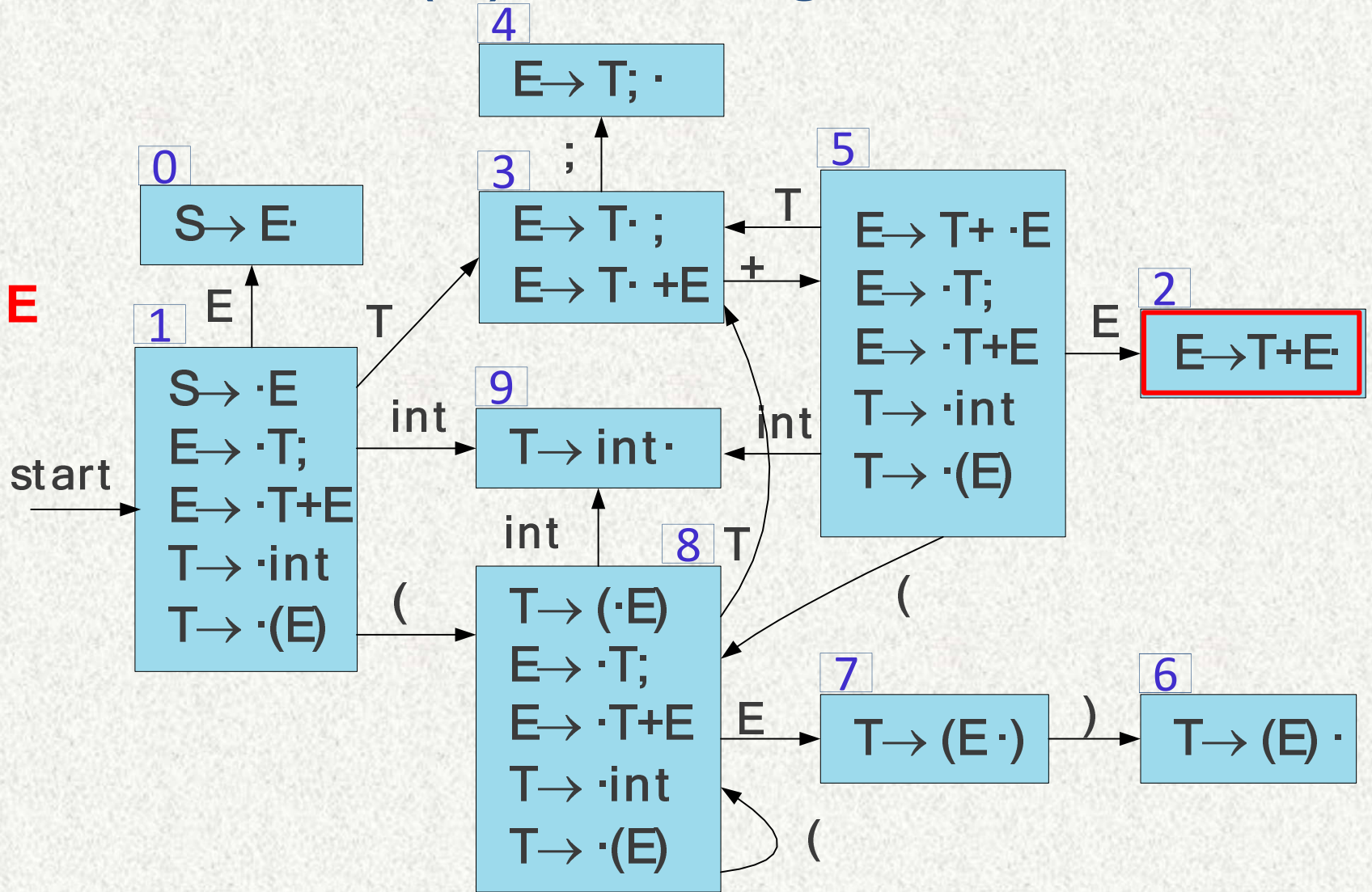
LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



LR(0) Parsing

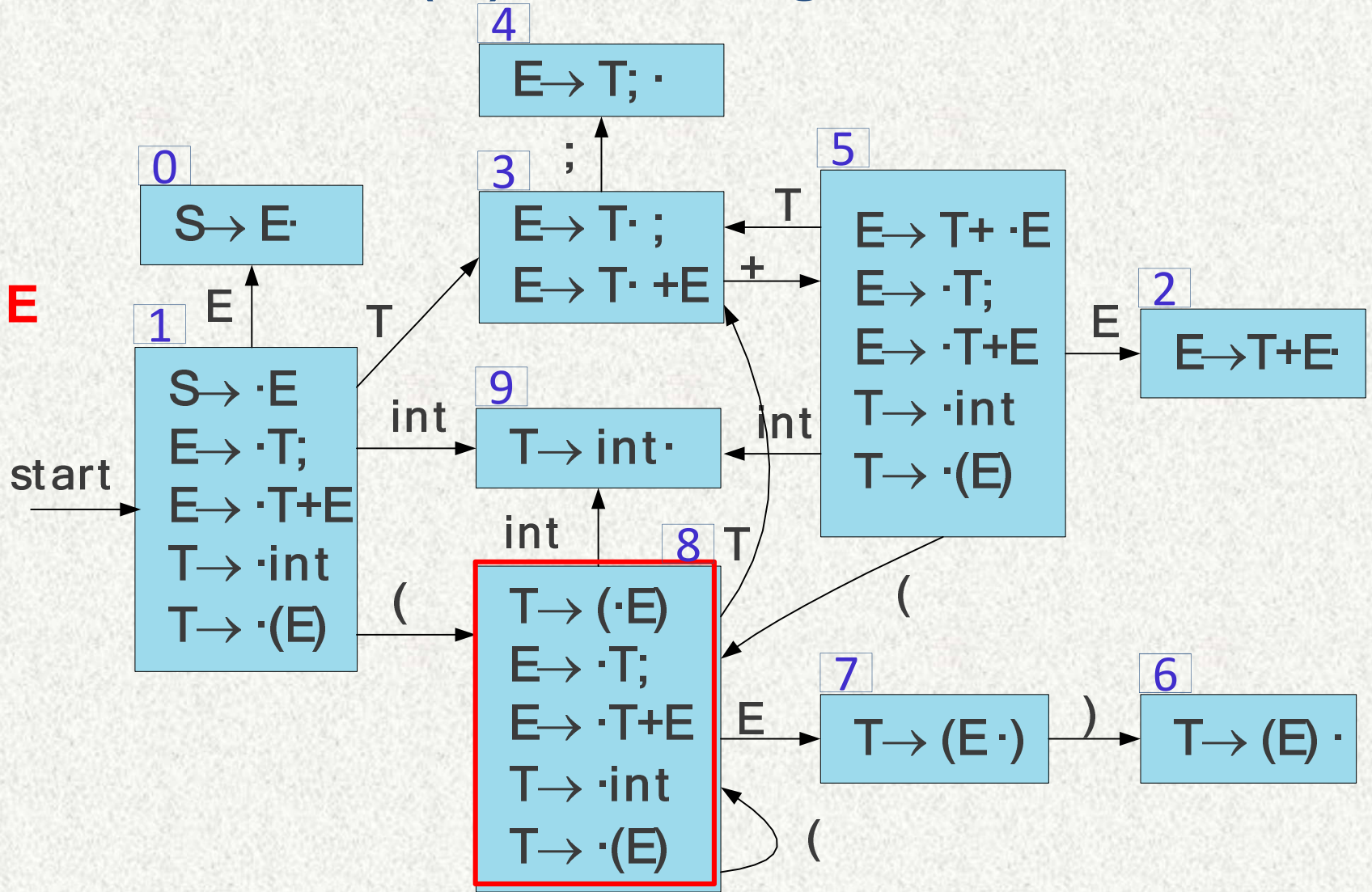
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



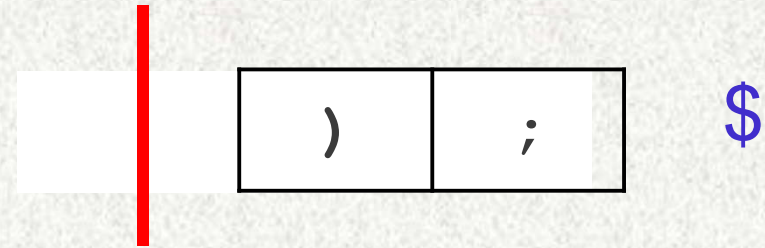
\$	T	+	(T	+	E)	;	\$
0	3	5	8	3	5	2				

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

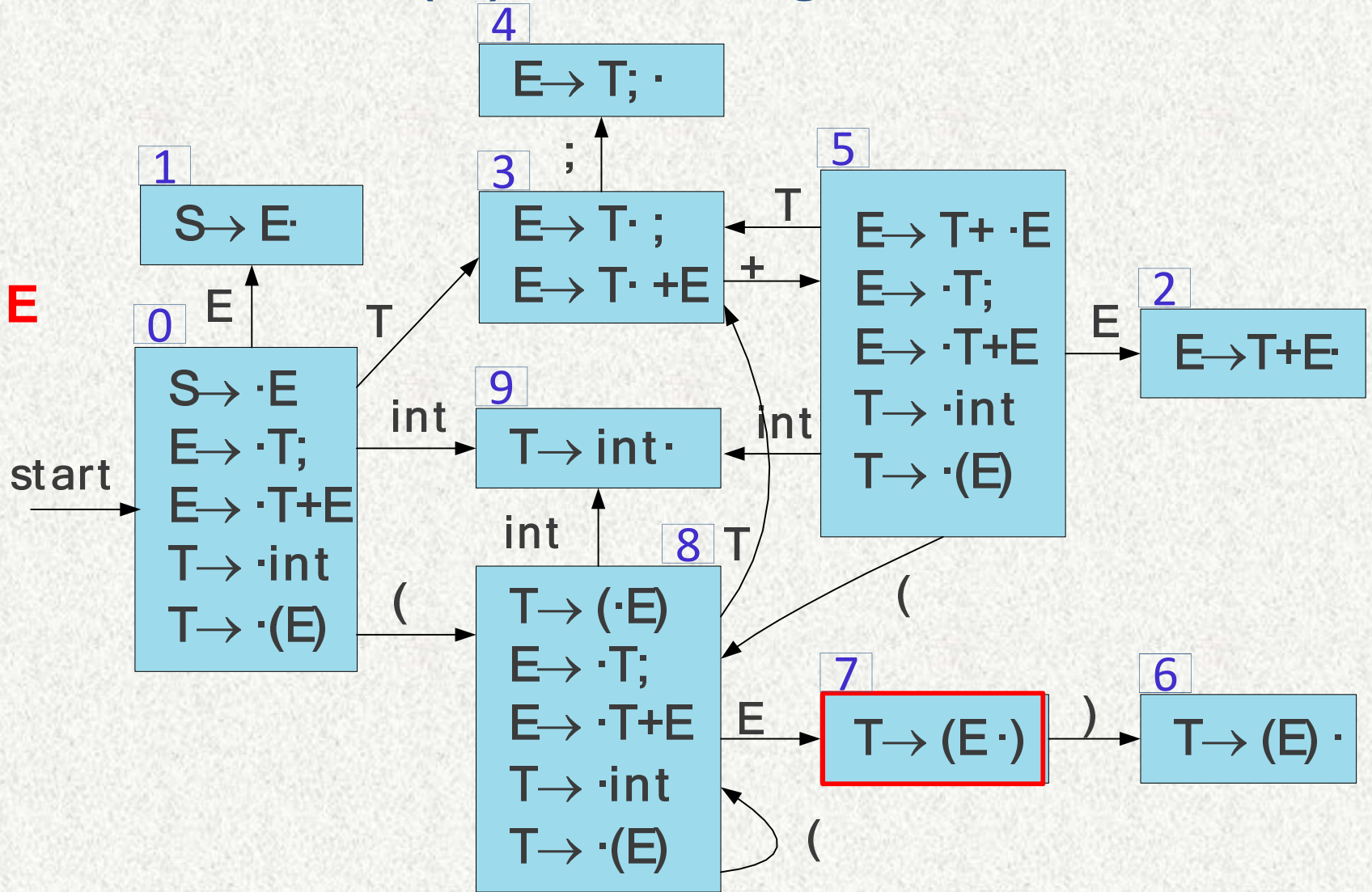


\$	T	+	(
0	3	5	8	



LR(0) Parsing

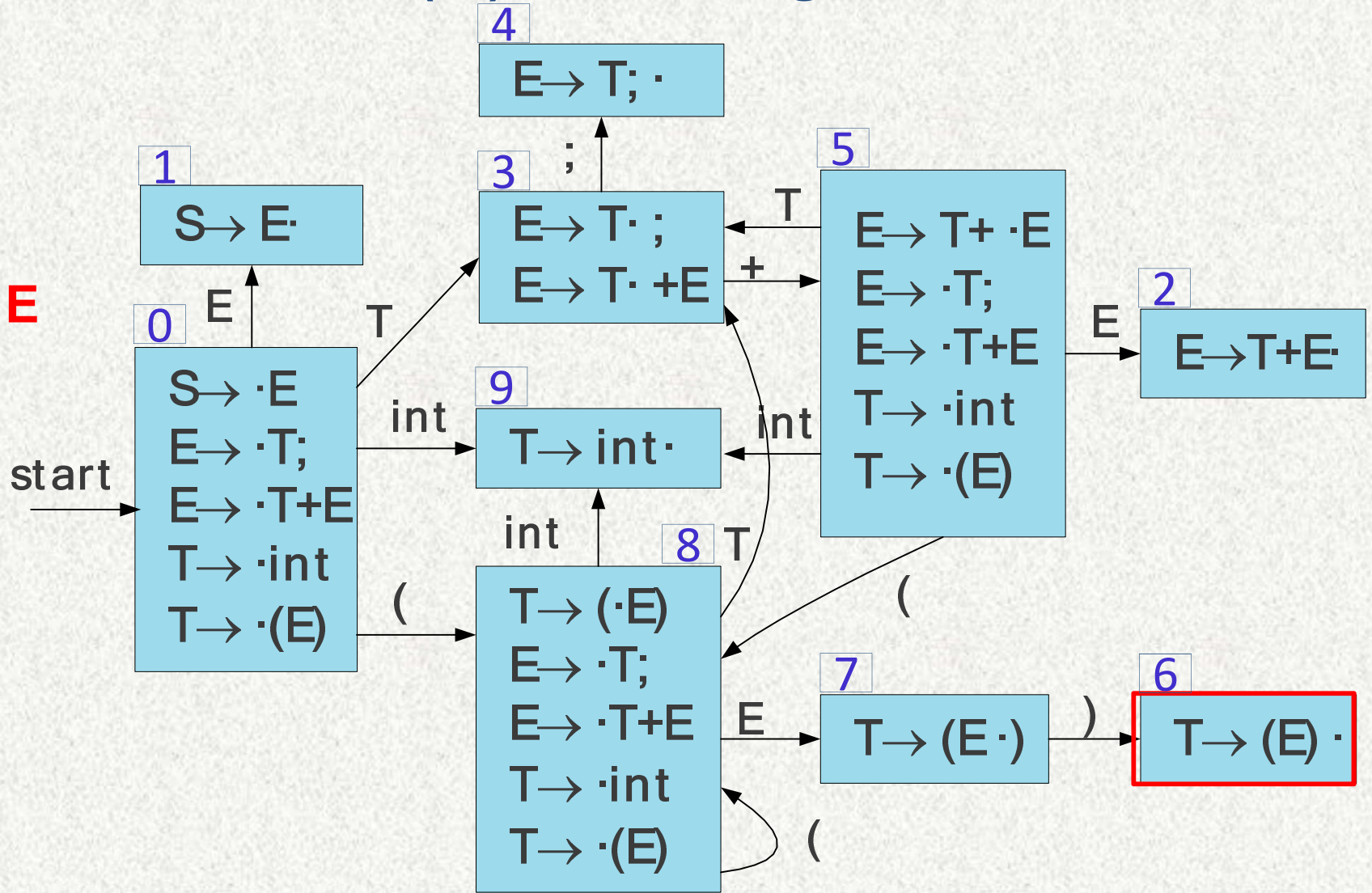
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



\$	T	+	(E)	;	\$
0	3	5	8	7				

LR(0) Parsing

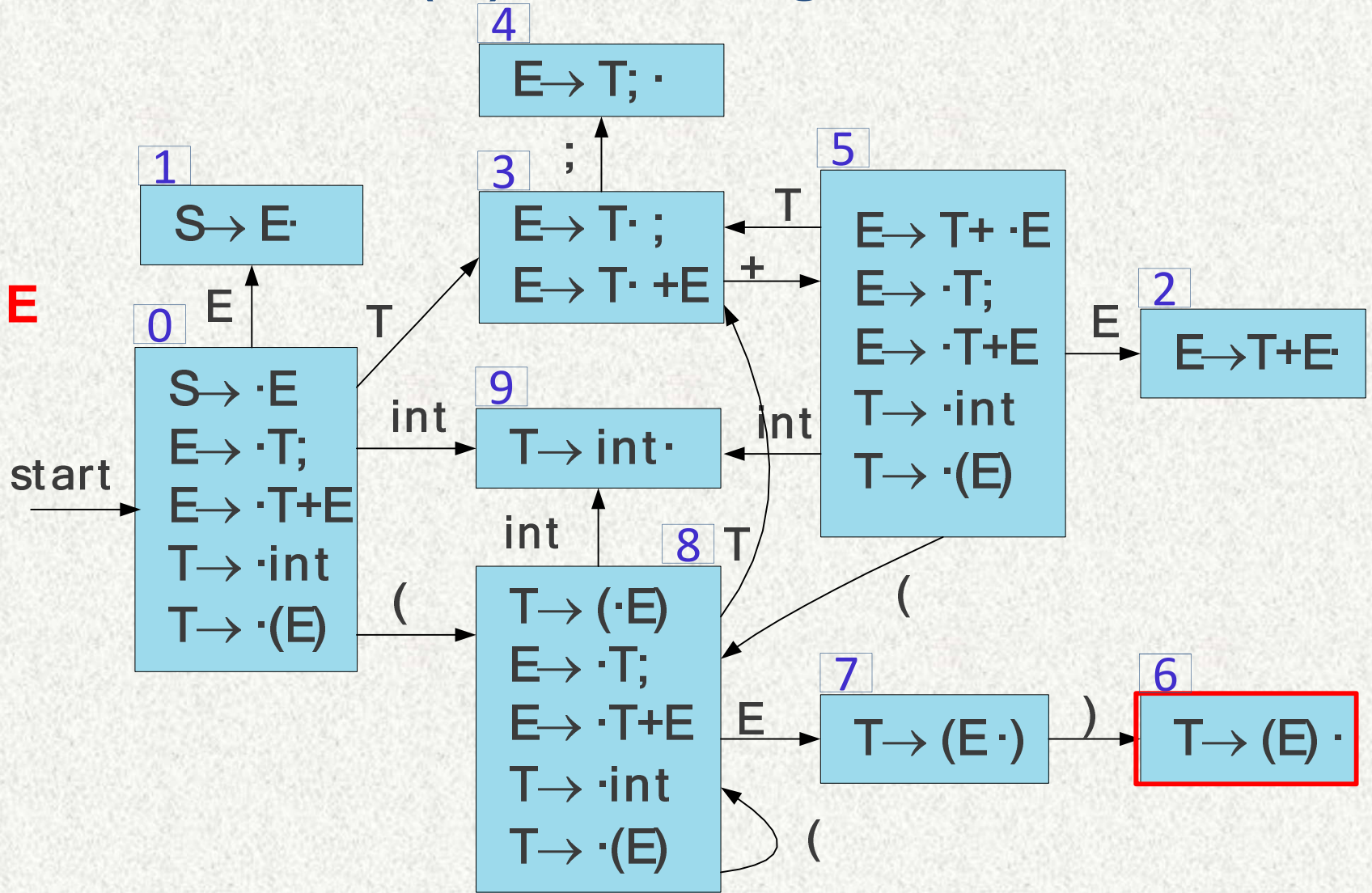
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



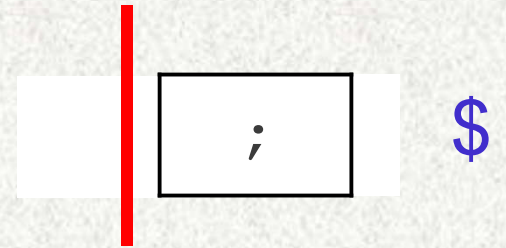
\$	T	+	(E)		;	\$
0	3	5	8	7	6			

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

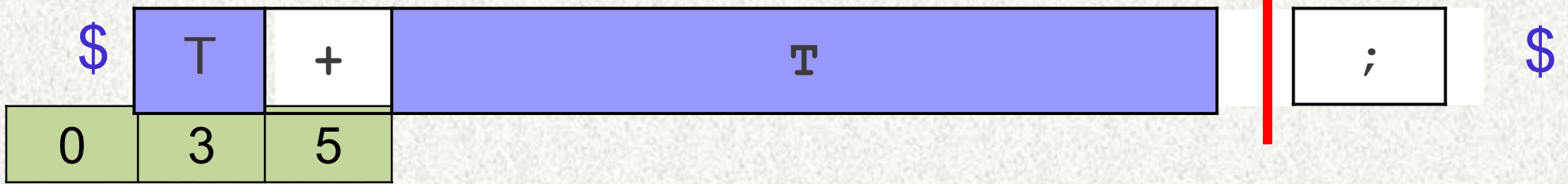
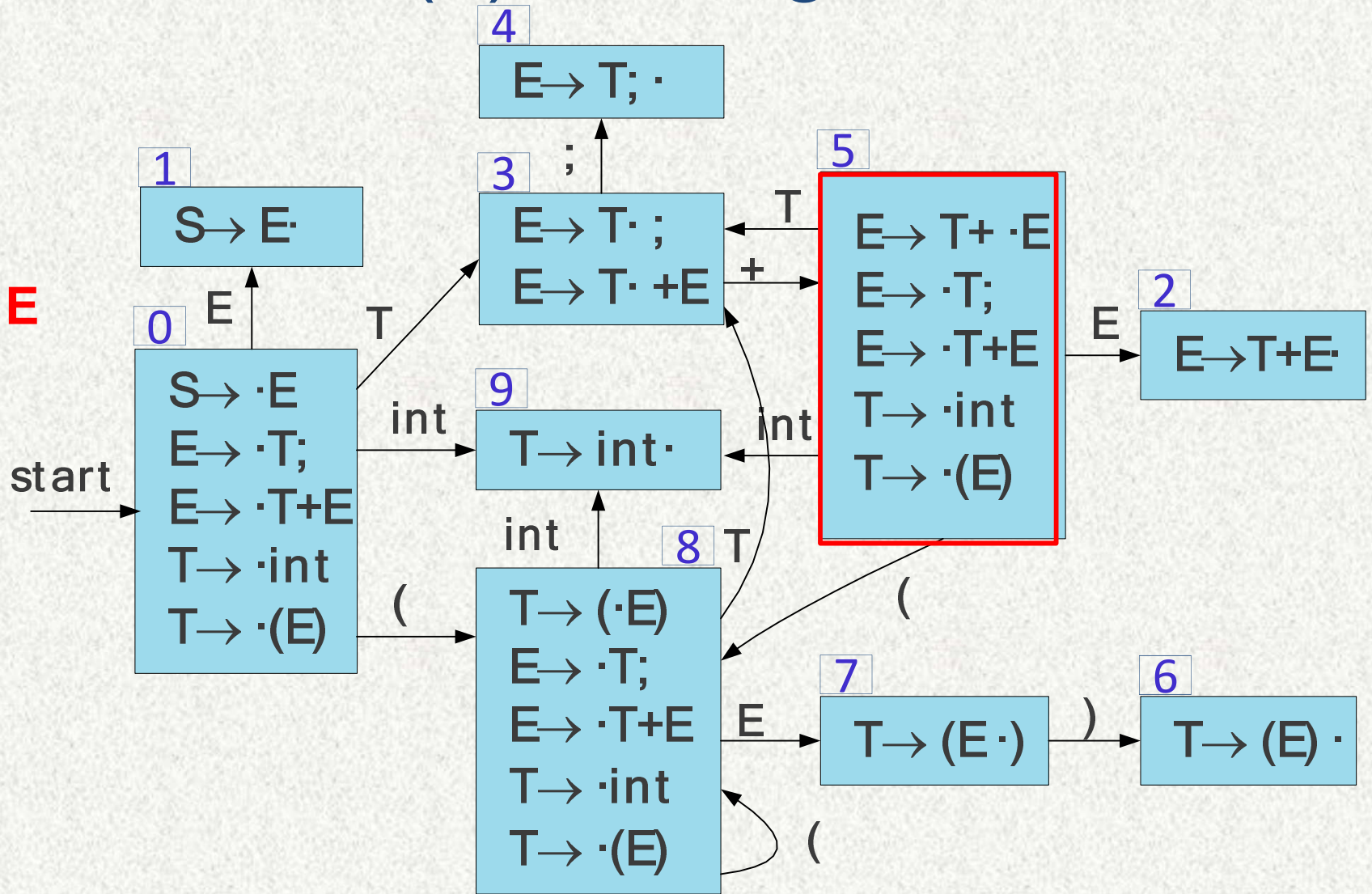


\$	T	+	
0	3	5	



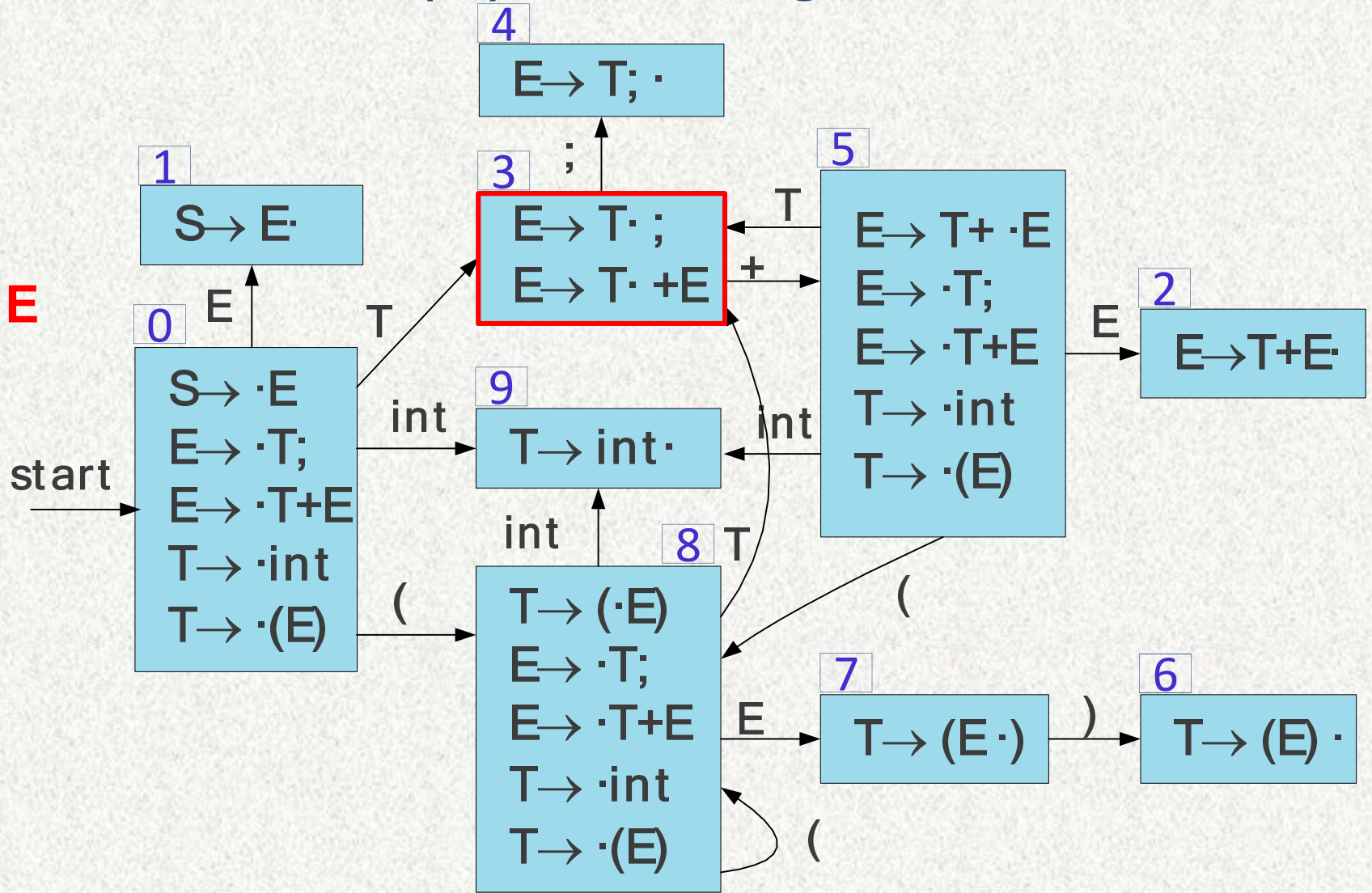
LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



LR(0) Parsing

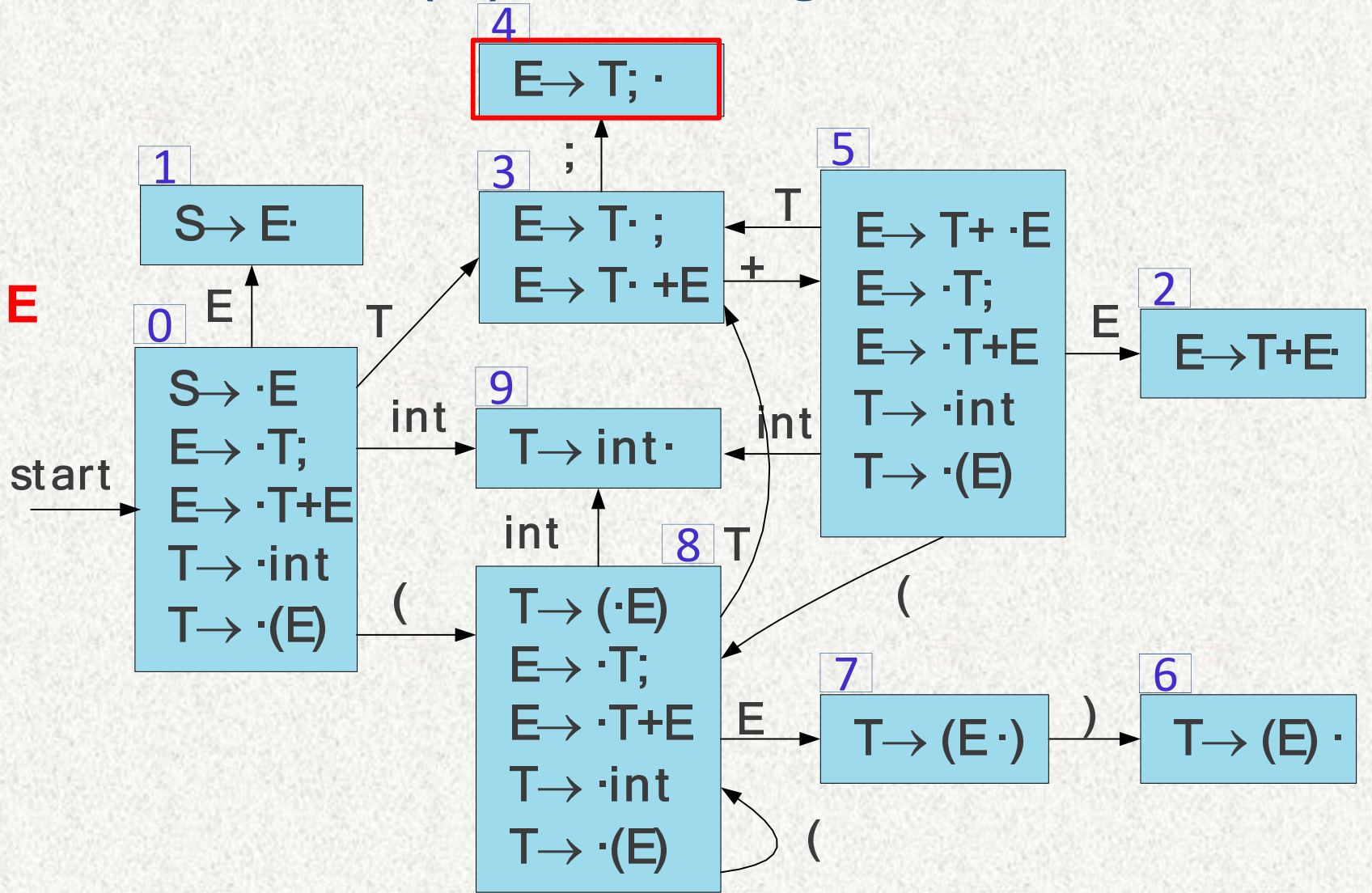
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



\$	T	+	T		;	\$
0	3	5	3			

LR(0) Parsing

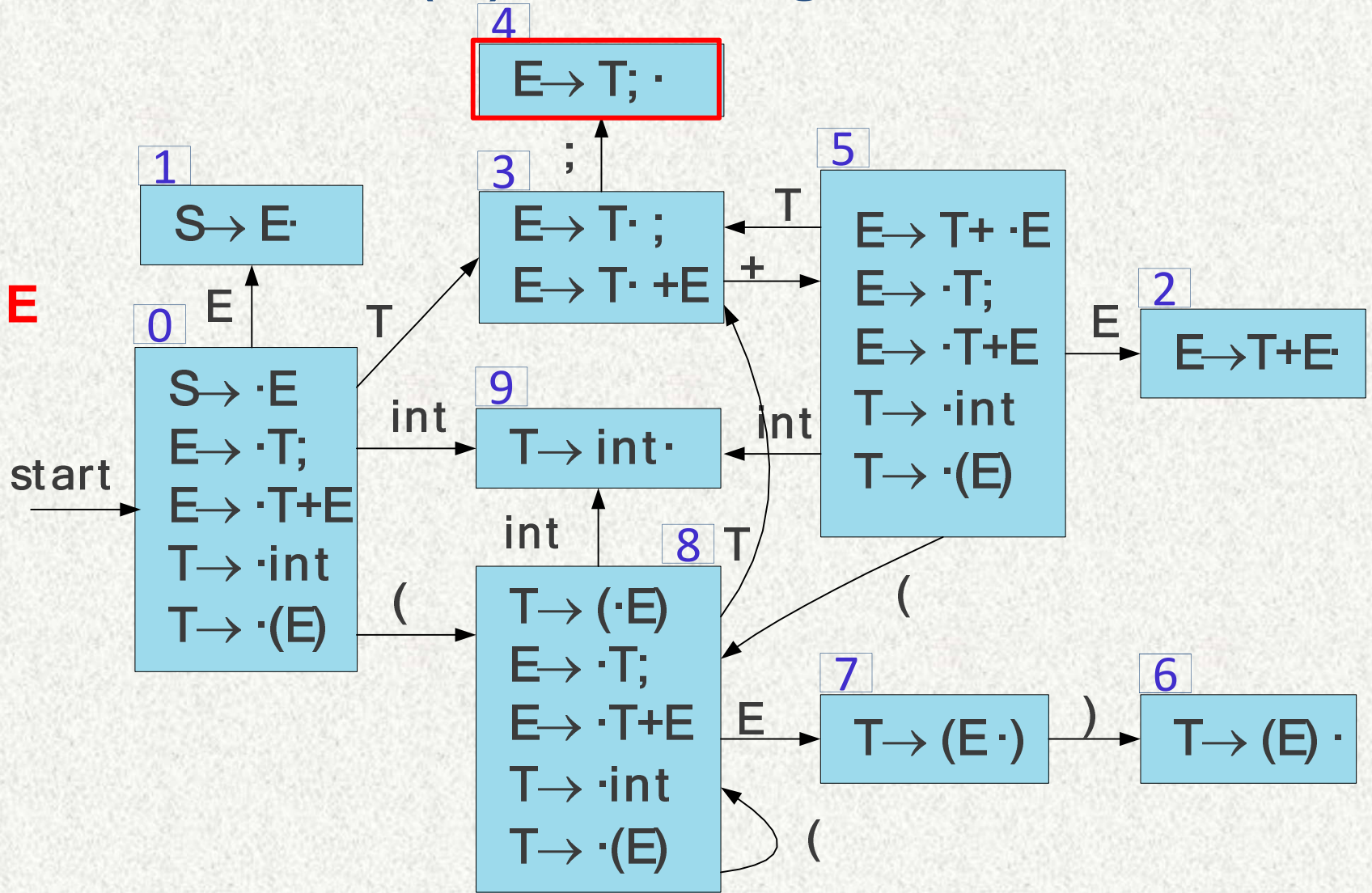
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



\$	T	+	T	;	\$
0	3	5	3	4	

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



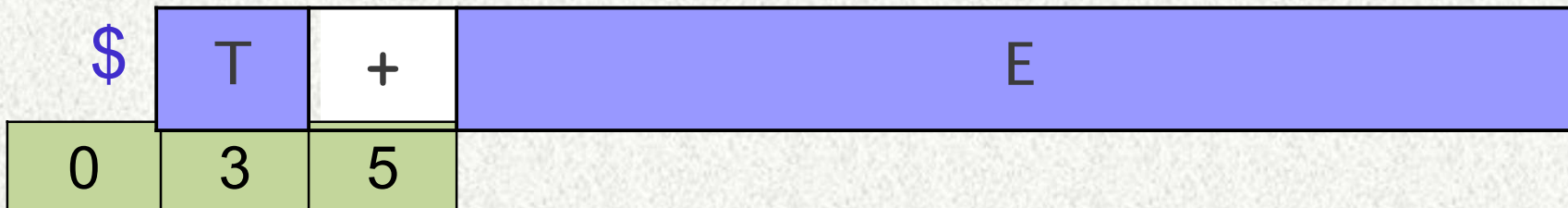
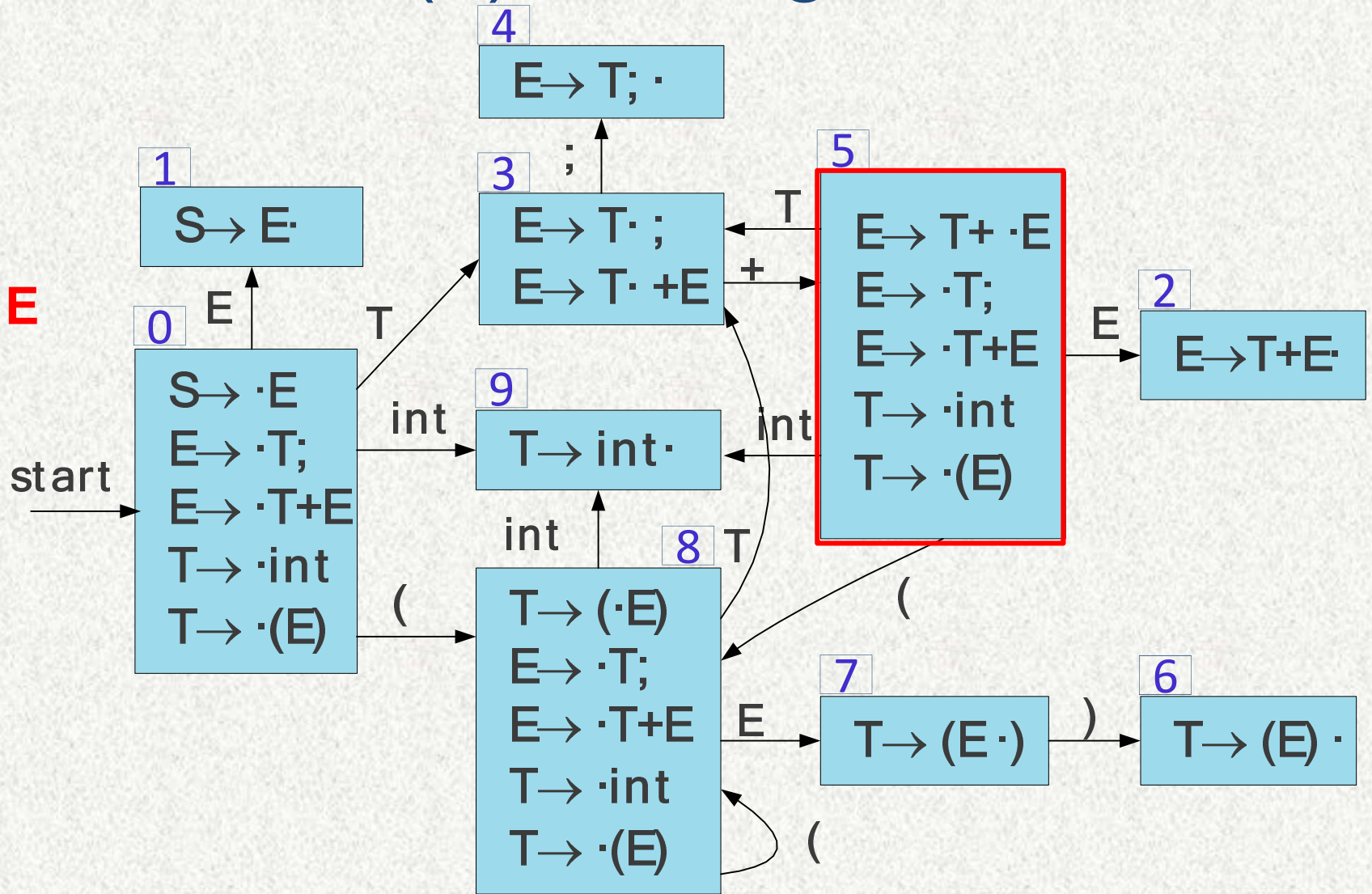
\$	T	+	
0	3	5	



\$

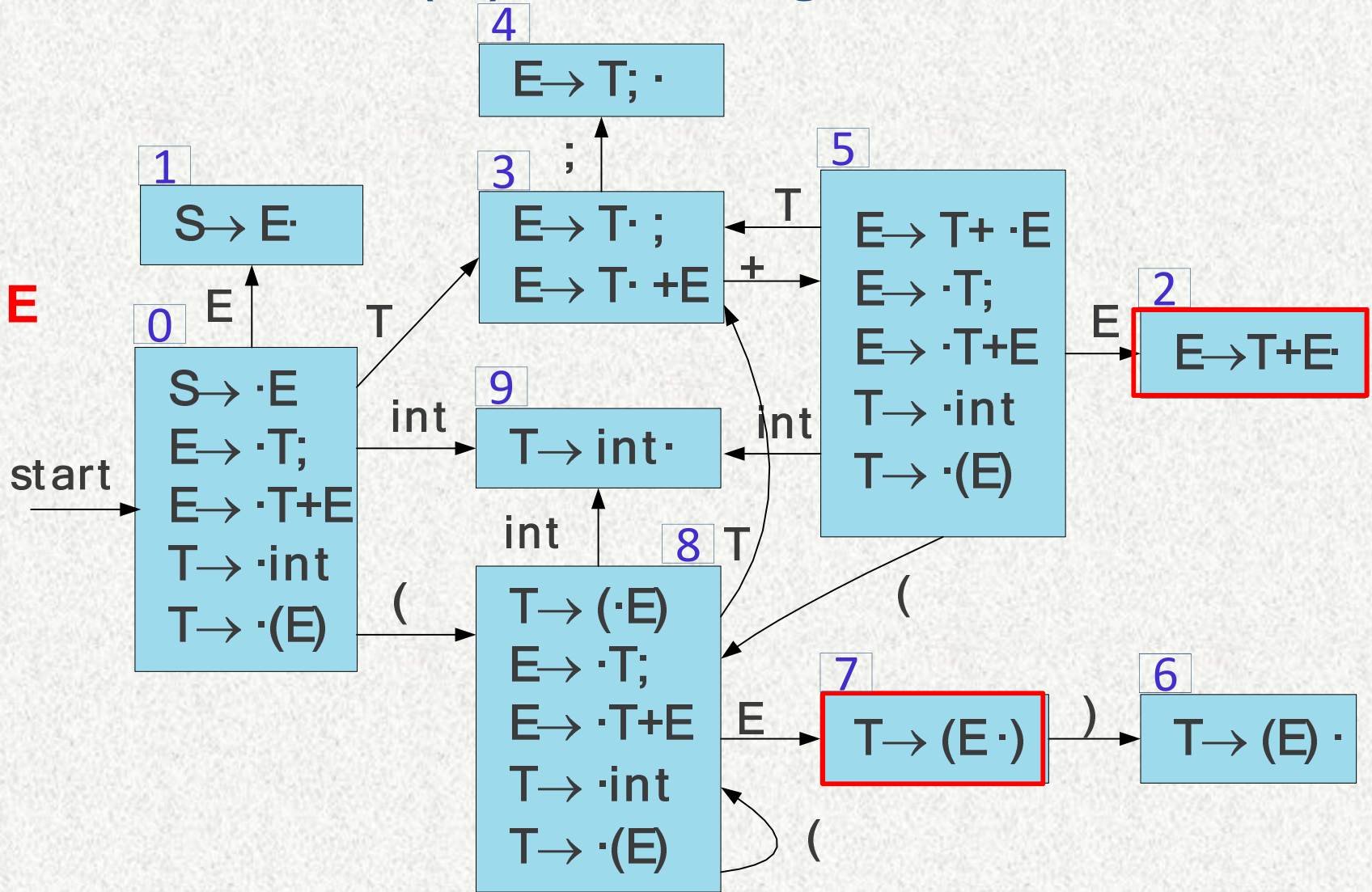
LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



LR(0) Parsing

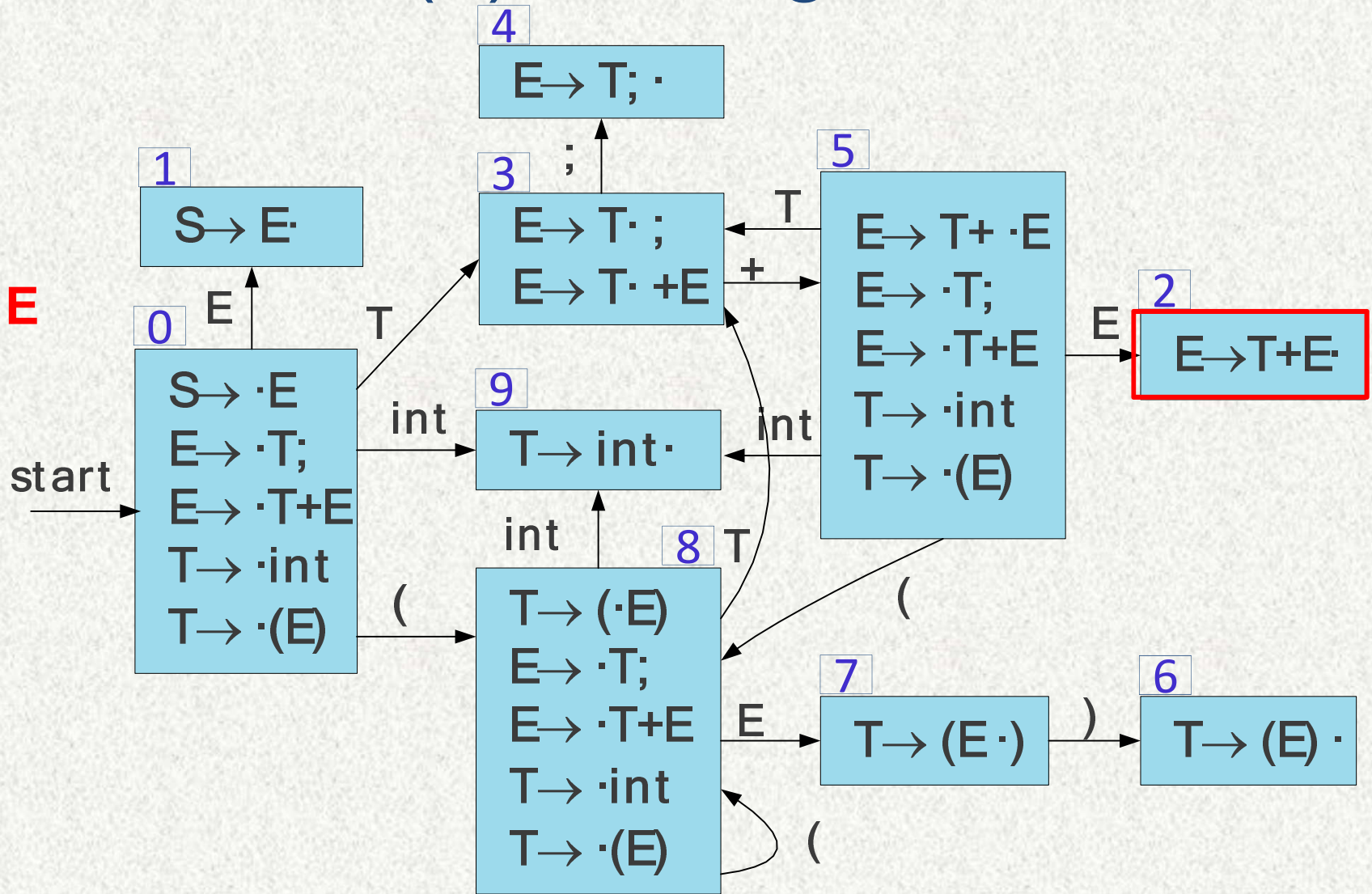
$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



\$	T	+	E	\$
0	3	5	2	

LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



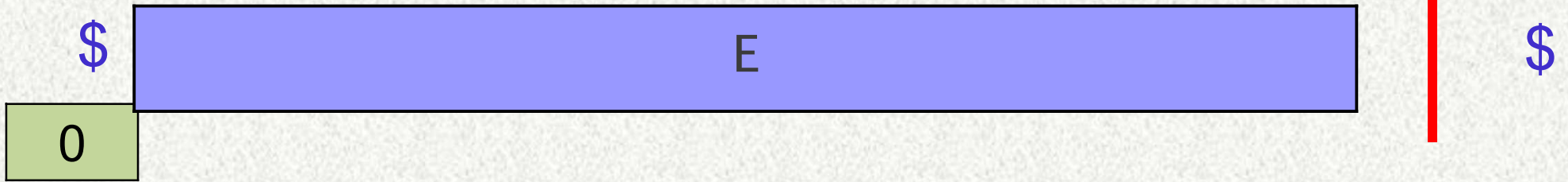
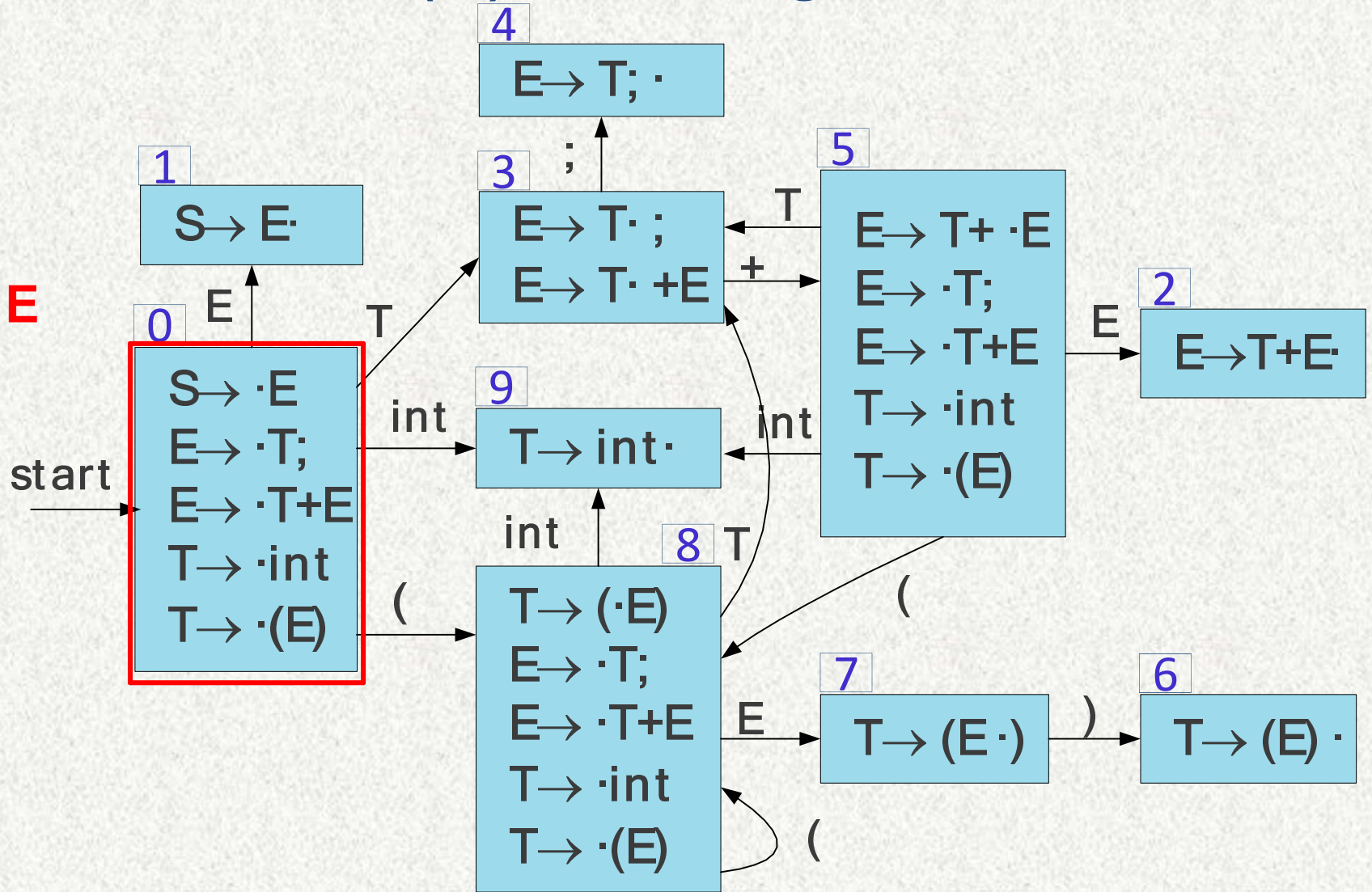
\$

\$

0

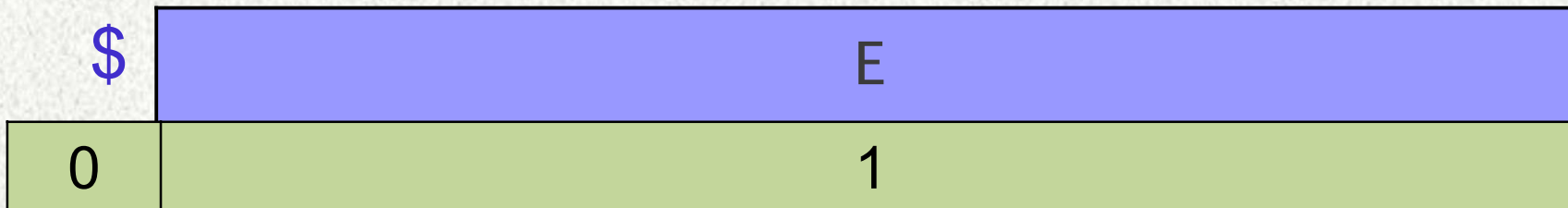
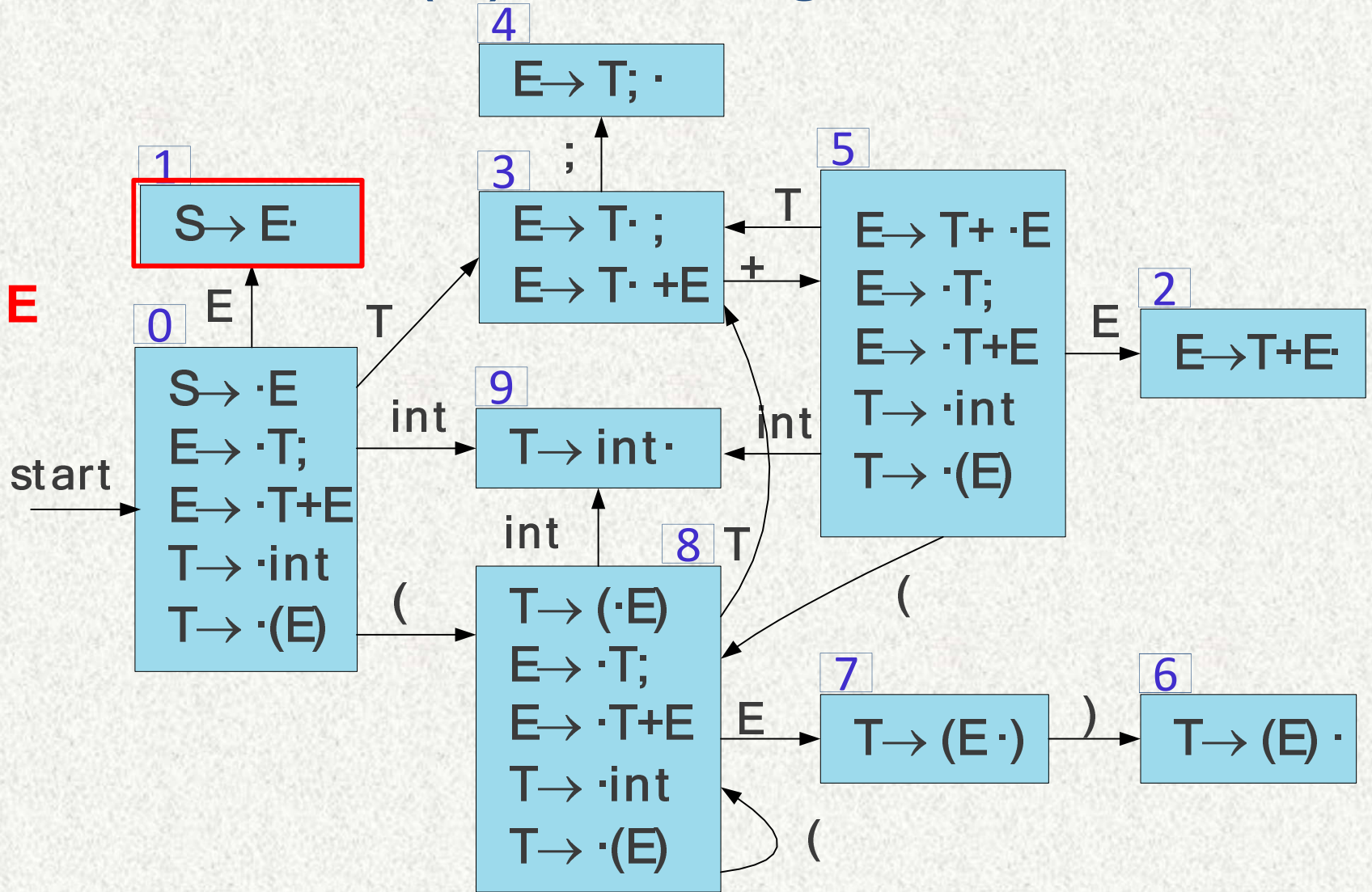
LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



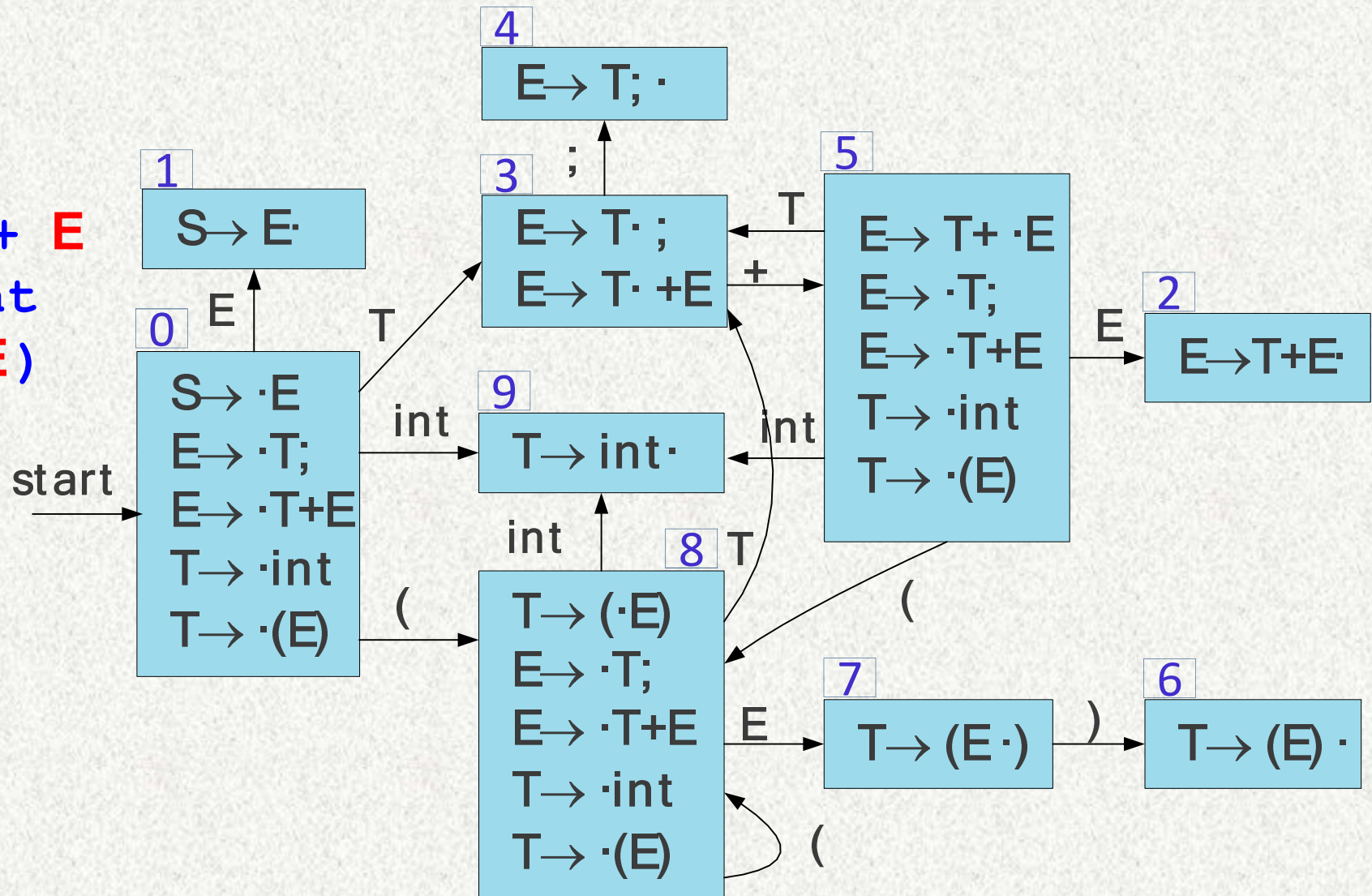
LR(0) Parsing

$S \rightarrow E$
 $E \rightarrow T;$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$



Building LR(0) Tables

1. $S \rightarrow E$
2. $E \rightarrow T;$
3. $E \rightarrow T + E$
4. $T \rightarrow \text{int}$
5. $T \rightarrow (E)$



LR Tables

state	int	+	;	()	E	T	\$	Action
0	9			8		1	3		Shift
1								acc	Accept
2									Reduce $E \rightarrow T + E$
3		5	4						Shift
4									Reduce $E \rightarrow T ;$
5	9			8		2	3		Shift
6									Reduce $T \rightarrow (E)$
7					6				Shift
8	9			8		7	3		Shift
9									Reduce $T \rightarrow int$

Representing the Automaton

- The ACTION function takes as arguments a state i and a terminal a (or $\$$, the input endmarker). The value of ACTION[i , a] can have one of four forms:
 - a) Shift j , where j is a state. The action taken by the parser effectively shifts input a to the stack, but uses state j to represent a .
 - b) Reduce $A \rightarrow \beta$. The action of the parser effectively reduces β on the top of the stack to head A .
 - c) Accept. The parser accepts the input and finishes parsing;
 - d) Error.
- We extend the GOTO function, defined on sets of items, to states: if GOTO [I_i , A] = I_j , then GOTO also maps a state i and a nonterminal A to state j .



Limit of LR(0)

LR Conflicts

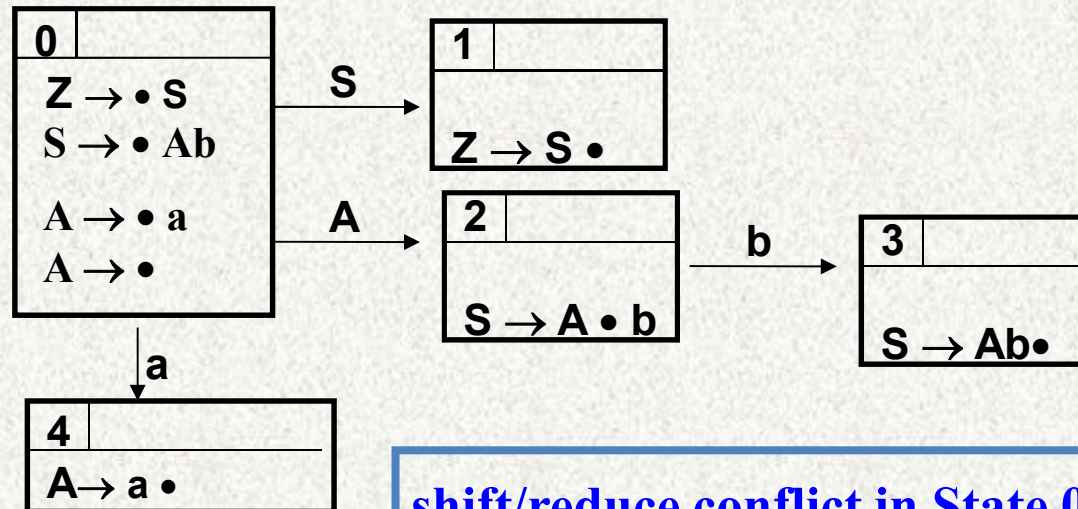
A **shift/reduce conflict** is an error where a shift/reduce parser cannot tell whether to shift a token or perform a reduction.

A **reduce/reduce conflict** is an error where a shift/reduce parser cannot tell which of many reductions to perform.

A grammar whose handle-finding automaton contains a shift/reduce conflict or a reduce/reduce conflict is not LR(0).

shift/reduce conflict

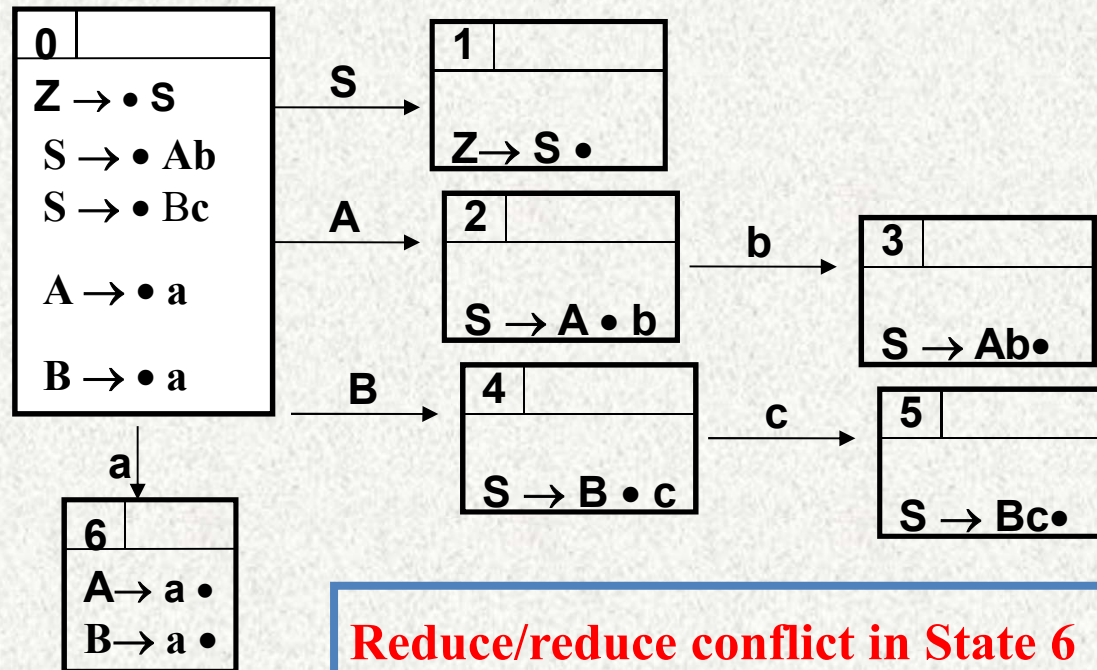
$V_T = \{a, b\}$
$V_N = \{S, A\}$
$S = S$
P: { (1) $S \rightarrow Ab$ (2) $A \rightarrow \varepsilon$ (3) $A \rightarrow a$ }



shift/reduce conflict in State 0
(1) Shift item: $A \rightarrow \bullet a$
(2) Reducible item: $A \rightarrow \bullet$

reduce/reduce conflict

$V_T = \{a, b, c\}$
$V_N = \{S, A, B\}$
$S = S$
P: {(1) $S \rightarrow Ab$ (2) $S \rightarrow Bc$ (3) $A \rightarrow a$ (4) $B \rightarrow a$ }



Reduce/reduce conflict in State 6
 (1) Reduce item 1: $A \rightarrow a \bullet$
 (2) Reduce item 2: $B \rightarrow a \bullet$

How to resolve?

- **Improve LR(0)**
 - **SLR** – simple LR parser
 - **LR** – most general LR parser
 - **LALR** – intermediate LR parser



SLR Parser

SLR(1)

SLR(1), simple LR(1) parsing, **uses the DFA of sets of LR(0) items** as constructed in the previous section

SLR(1) increases the power of LR(0) parsing significantly by **using the next token** in the input string

- First, it **consults the input token *before*** a shift to make sure that an appropriate DFA transition exists
- Second, it **uses the **Follow set** of a non-terminal to decide if** a reduction should be performed

SLR(1)

- **Choose the action by looking ahead of a symbol**
 - For LR(0) itemset $I = \{X \rightarrow \gamma \bullet a \beta, A \rightarrow \pi \bullet, B \rightarrow \pi' \bullet\}$, denoted as state S_i :
 - Conflict in cell (S_i, a) : Reduce or shift?
 - What if $\text{Follow}(A) \cap \text{Follow}(B) = \Phi$, specifically, $a \notin \text{Follow}(A), a \notin \text{Follow}(B)$, what can we do?

SLR(1)

- **Choose the action by looking ahead of a symbol, for cell (S_i, a)**
 - S/R conflict:
 - Choose shift: if there exist $A \rightarrow \alpha \bullet a \beta$
 - Choose reduce: if there exist $B \rightarrow \pi \bullet$, and $a \in \text{follow}(B)$
 - R/R conflict
 - Choose reduce with P1: if there exist $A \rightarrow \pi \bullet$, $a \in \text{follow}(A)$, where $P1 = A \rightarrow \pi$
 - Choose reduce with P2, if there exist $B \rightarrow \pi' \bullet$, $a \in \text{follow}(B)$, where $P2 = B \rightarrow \pi'$

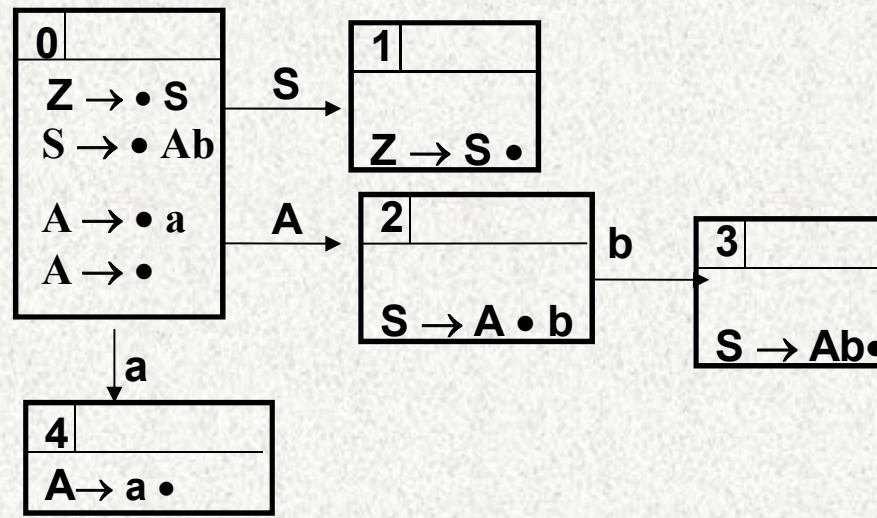
LR(0) table 1 with S/R conflict

$V_T = \{a, b\}$

 $V_N = \{S, A\}$

 $S = S$

P:
{ (1) $S \rightarrow Ab$
(2) $A \rightarrow \epsilon$
(3) $A \rightarrow a$
}



In state 0:
(1) shift: $A \rightarrow \bullet a$
(2) reduce: $A \rightarrow \bullet$

	Action			Goto	
	a	b	#	S	A
0	S4;R2	R2	R2	1	3
1			Accept		
2		S3			
3	R1	R1	R1		
4	R3	R3	R3		

LR(0) table 1 **without** S/R conflict

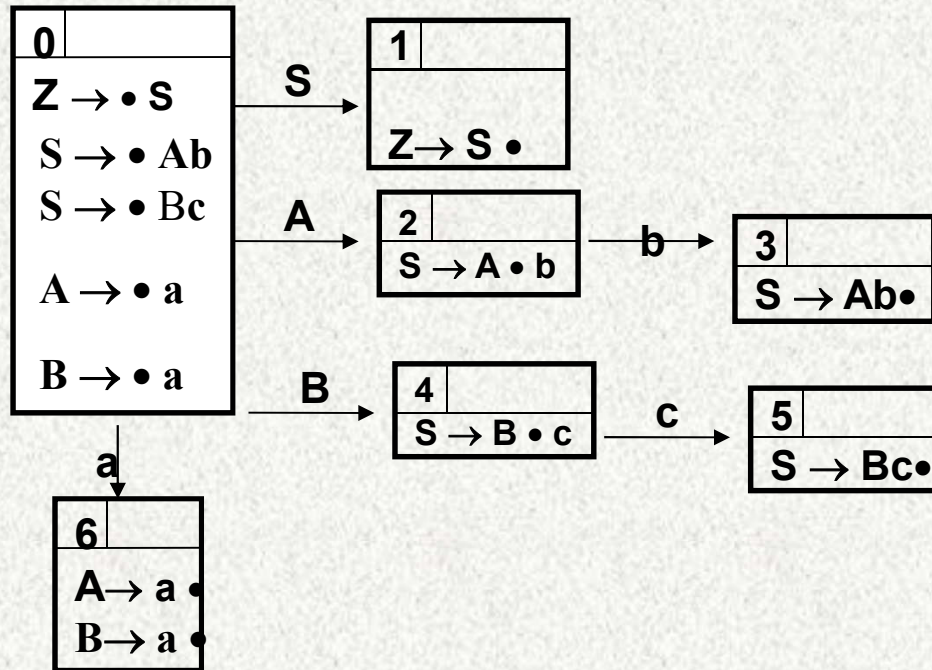
$V_T = \{a, b\}$
$V_N = \{S, A\}$
$S = S$
P: { (1) $S \rightarrow Ab$ (2) $A \rightarrow \varepsilon$ (3) $A \rightarrow a$ }

	Action			Goto	
	a	b	#	S	A
0	S4	R2		1	3
1			Accept		
2		S3			
3			R1		
4		R3			

Resolve conflict with follow(A)

LR(0) table 2 with S/R conflict

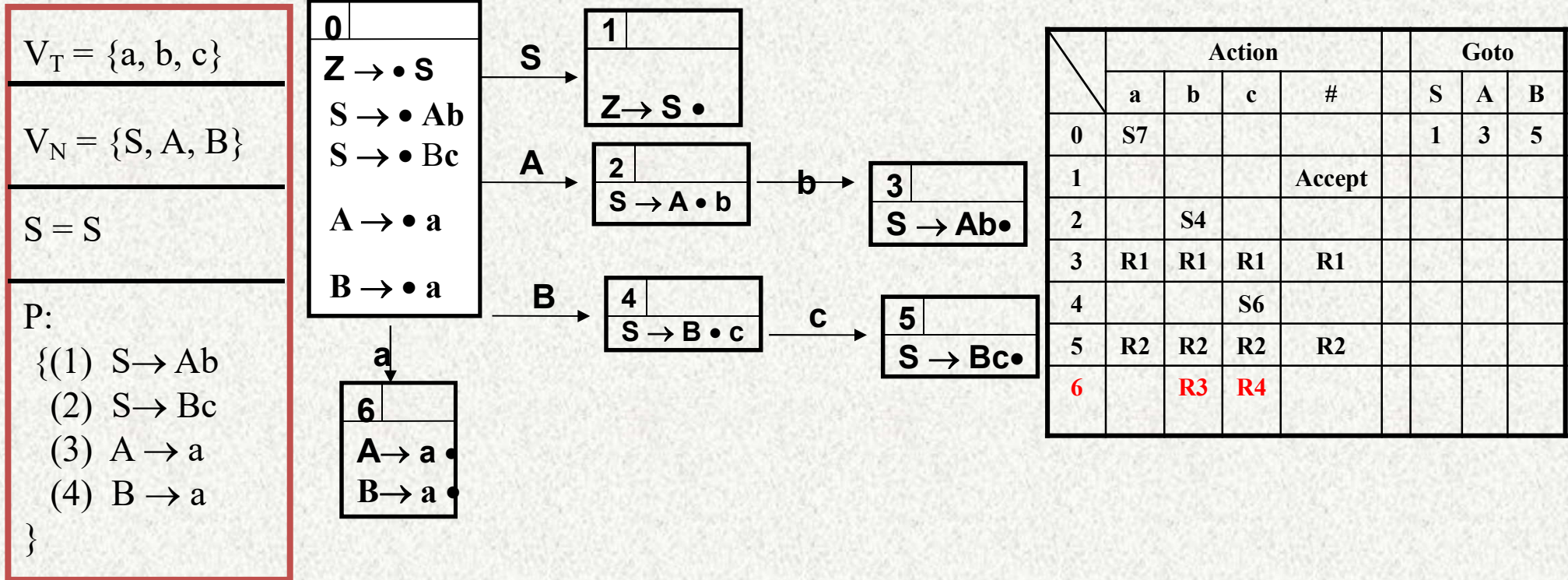
$V_T = \{a, b, c\}$
 $V_N = \{S, A, B\}$
 $S = S$
 P:
 {(1) $S \rightarrow Ab$
 (2) $S \rightarrow Bc$
 (3) $A \rightarrow a$
 (4) $B \rightarrow a$
 }



	Action				Goto		
	a	b	c	#	S	A	B
0	S7				1	3	5
1				Accept			
2		S4					
3	R1	R1	R1	R1			
4			S6				
5	R2	R2	R2	R2			
6	R3 R4	R3 R4	R3 R4	R3 R4			

Reduce/reduce conflict in State 6
(1) Reduce item 1: $A \rightarrow a \bullet$
(2) Reduce item 2: $B \rightarrow a \bullet$

LR(0) table 2 **without** S/R conflict



	Action				Goto		
	a	b	c	#	S	A	B
0	S7				1	3	5
1				Accept			
2		S4					
3	R1	R1	R1	R1			
4			S6				
5	R2	R2	R2	R2			
6		R3	R4				

Reduce/reduce conflict in State 6

(1) Reduce item 1: $A \rightarrow a \bullet$

(2) Reduce item 2: $B \rightarrow a \bullet$

Resolve conflict with follow(A) and follow(B)

Limitation of SLR(1)

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a :
 - if the $A \rightarrow \alpha$ is in the I_i and a is $FOLLOW(A)$
- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta \alpha$ and the state i are on the top stack.
- This means that making reduction in this case is not correct.

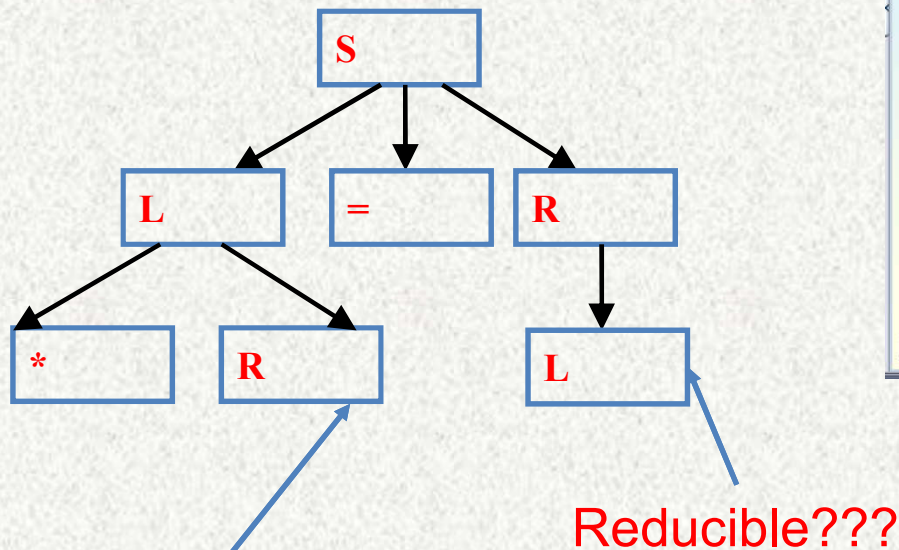
Limitation of SLR(1)

Example 4.51: Let us reconsider Example 4.48, where in state 2 we had item $R \rightarrow L\cdot$, which could correspond to $A \rightarrow \alpha$ above, and a could be the = sign, which is in $\text{FOLLOW}(R)$. Thus, the SLR parser calls for reduction by $R \rightarrow L$ in state 2 with = as the next input (the shift action is also called for, because of item $S \rightarrow L\cdot=R$ in state 2). However, there is no right-sentential form of the grammar in Example 4.48 that begins $R = \dots$. Thus state 2, which is the state corresponding to viable prefix L only, should not really call for reduction of that L to R . \square

Limitation of SLR(1)

- 1. $S' \rightarrow S$ 2. $S \rightarrow \underline{L=R}$
- 3. $S \rightarrow R$ 4. $L \rightarrow \underline{*R}$
- 5. $L \rightarrow i$ 6. $R \rightarrow \underline{L}$

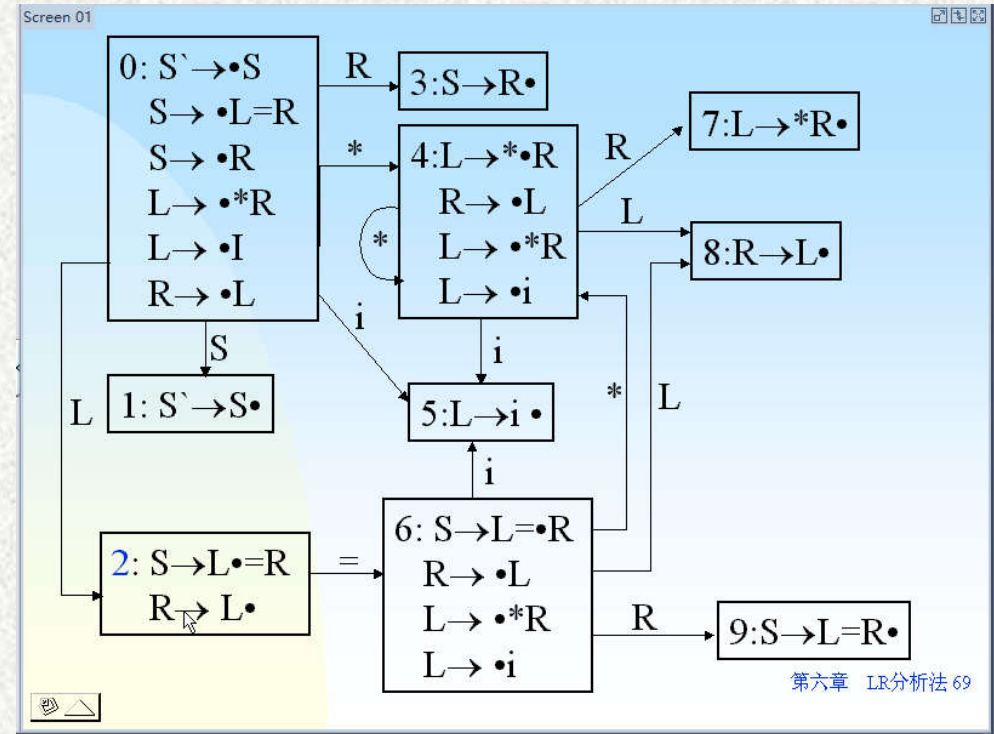
$\text{follow}(R) = \{\#, =\}$



Follow symbol "=" is actually from here

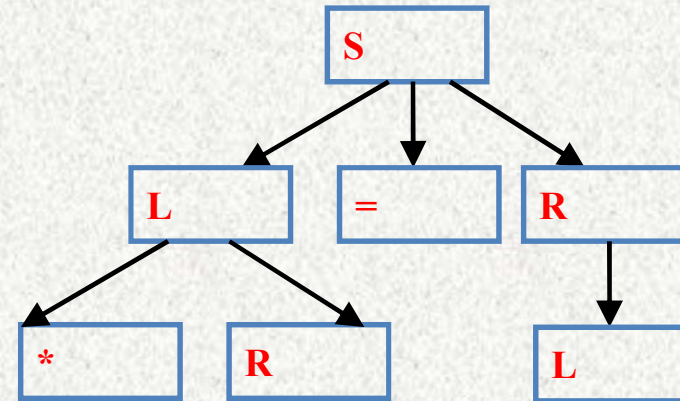
How exactly does "R=" come from: $S' \Rightarrow L=R \Rightarrow *R=R$

We must have a * before R.



Limitation of SLR(1)

- 1. $S' \rightarrow S$ 2. $S \rightarrow \underline{L=R}$
- 3. $S \rightarrow R$ 4. $L \rightarrow \underline{*R}$
- 5. $L \rightarrow i$ 6. $R \rightarrow \underline{L}$



Solution: LR(1), not consider **ALL follow** symbols, instead, we consider **all feasible follow symbols**

To avoid some of invalid reductions, the states need to carry more information. Extra information is put into a state by including a terminal symbol as a second component in an item.

Homework

Page 258: 4.6.2, 4.6.3;

Page 258: 4.6.4 --- answer the question for 4.2.2(d) (f);



LR(1) Parser

LR(1) Item

- A LR(1) item is: $A \rightarrow \alpha.\beta, \mathbf{a}$,
where \mathbf{a} is the look-ahead of the LR(1) item (\mathbf{a} is a terminal or end-marker.)

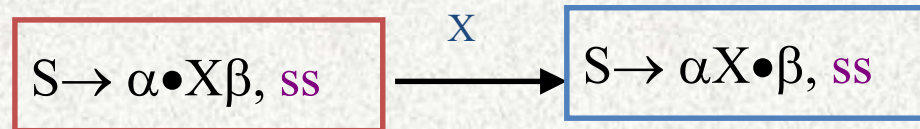
Constructing LR(1) automaton

```
SetOfItems CLOSURE(I) {  
    repeat  
        for ( each item [ $A \rightarrow \alpha \cdot B \beta, a$ ] in I )  
            for ( each production  $B \rightarrow \gamma$  in  $G'$  )  
                for ( each terminal  $b$  in FIRST( $\beta a$ ) )  
                    add [ $B \rightarrow \cdot \gamma, b$ ] to set I;  
    until no more items are added to I;  
    return I;  
}  
  
SetOfItems GOTO(I, X) {  
    initialize J to be the empty set;  
    for ( each item [ $A \rightarrow \alpha \cdot X \beta, a$ ] in I )  
        add item [ $A \rightarrow \alpha X \cdot \beta, a$ ] to set J;  
    return CLOSURE(J);  
}  
  
void items( $G'$ ) {  
    initialize C to CLOSURE( $\{[S' \rightarrow \cdot S, \$]\}$ );  
    repeat  
        for ( each set of items I in C )  
            for ( each grammar symbol X )  
                if ( GOTO(I, X) is not empty and not in C )  
                    add GOTO(I, X) to C;  
    until no new sets of items are added to C;  
}
```

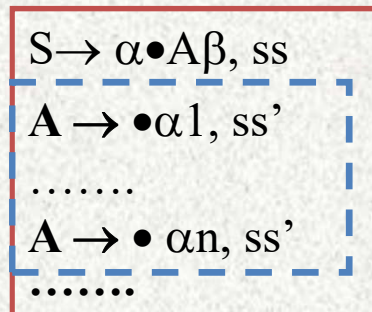
Key about look-ahead symbols

$$S_0 = \text{CLOSURE}(\{(S' \rightarrow \bullet S, \{\#\})\})$$

- **Type 1**



- **Type 2**



$ss' = \text{first}(\beta)$, if β does not derive empty;

$ss' = (\text{first}(\beta) - \{\epsilon\}) \cup ss$, if β derives empty;

An Example

1. $S' \rightarrow S$
2. $S \rightarrow C C$
3. $C \rightarrow c C$
4. $C \rightarrow d$

$$I_0: \text{closure}(\{(S' \rightarrow \bullet S, \$)\}) =$$

- $(S' \rightarrow \bullet S, \$)$
- $(S \rightarrow \bullet C C, \$)$
- $(C \rightarrow \bullet c C, c/d)$
- $(C \rightarrow \bullet d, c/d)$

$$I_1: \text{goto}(I_0, S) = (S' \rightarrow S \bullet, \$)$$

$$I_2: \text{goto}(I_0, C) =$$

- $(S \rightarrow C \bullet C, \$)$
- $(C \rightarrow \bullet c C, \$)$
- $(C \rightarrow \bullet d, \$)$

$$I_3: \text{goto}(I_0, c) =$$

- $(C \rightarrow c \bullet C, c/d)$
- $(C \rightarrow \bullet c C, c/d)$
- $(C \rightarrow \bullet d, c/d)$

$$I_4: \text{goto}(I_0, d) =$$

- $(C \rightarrow d \bullet, c/d)$

$$I_5: \text{goto}(I_3, C) =$$

- $(S \rightarrow C C \bullet, \$)$

An Example

l_6 : goto(l_3 , c) =
(C \rightarrow c • C, \$)
(C \rightarrow • c C, \$)
(C \rightarrow • d, \$)

l_7 : goto(l_3 , d) =
(C \rightarrow d •, \$)

l_8 : goto(l_4 , C) =
(C \rightarrow c C •, c/d)

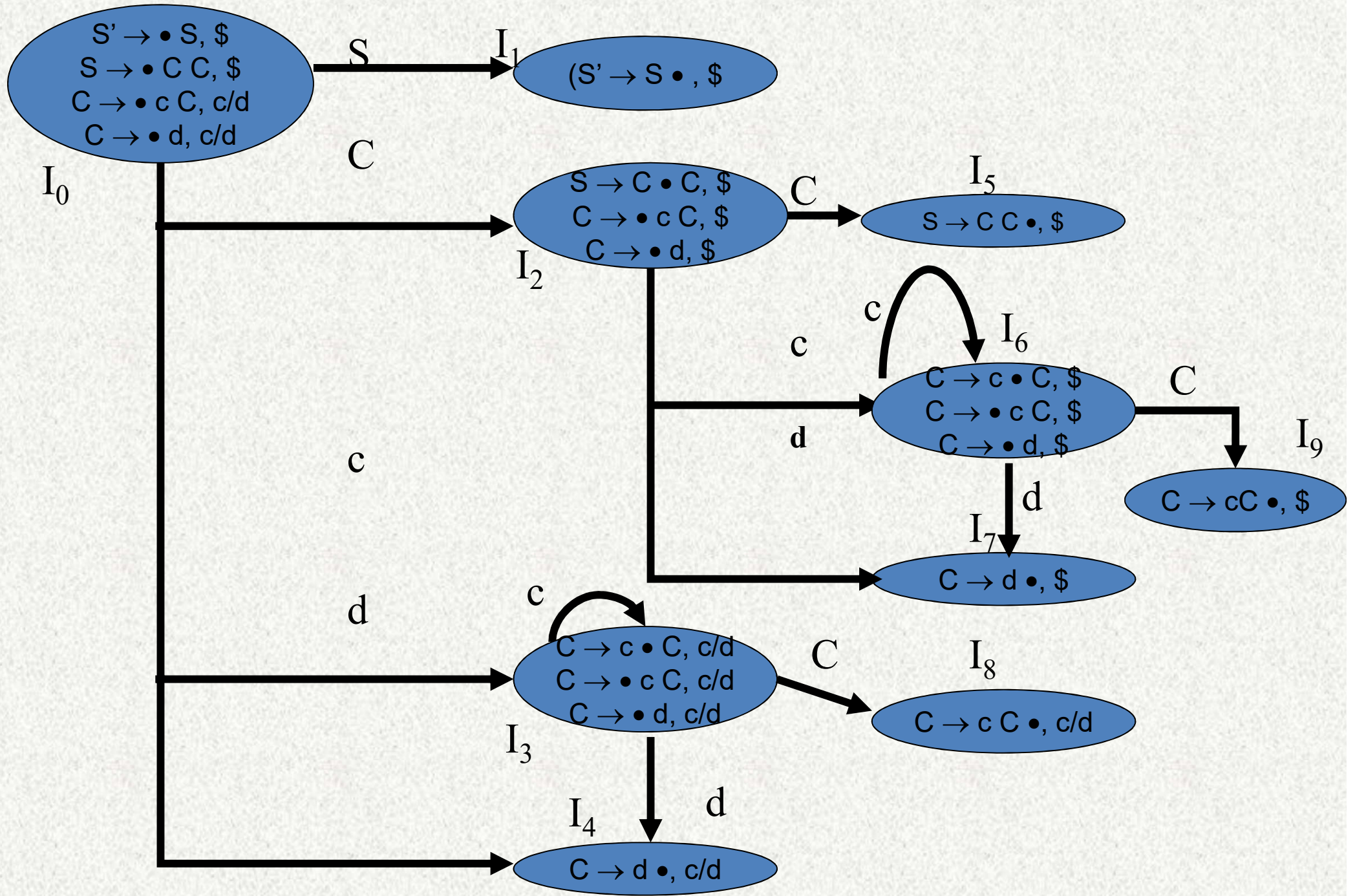
: goto(l_4 , c) = l_4

: goto(l_4 , d) = l_5

l_9 : goto(l_7 , c) =
(C \rightarrow c C •, \$)

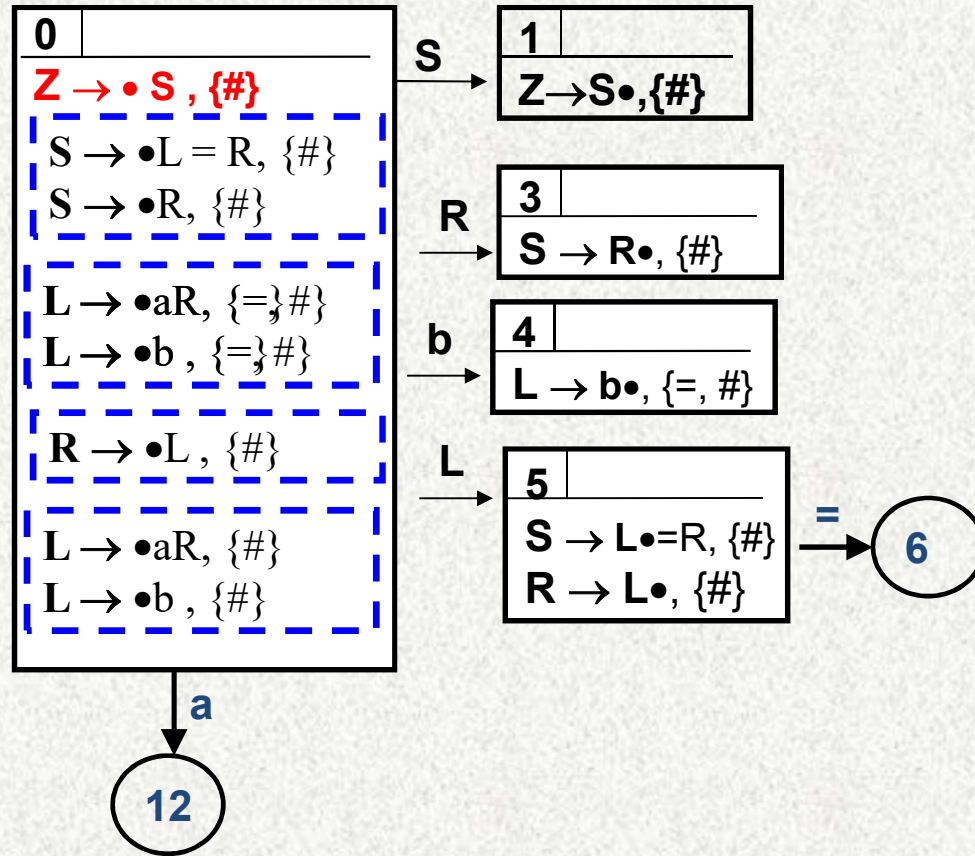
: goto(l_7 , c) = l_7

: goto(l_7 , d) = l_8

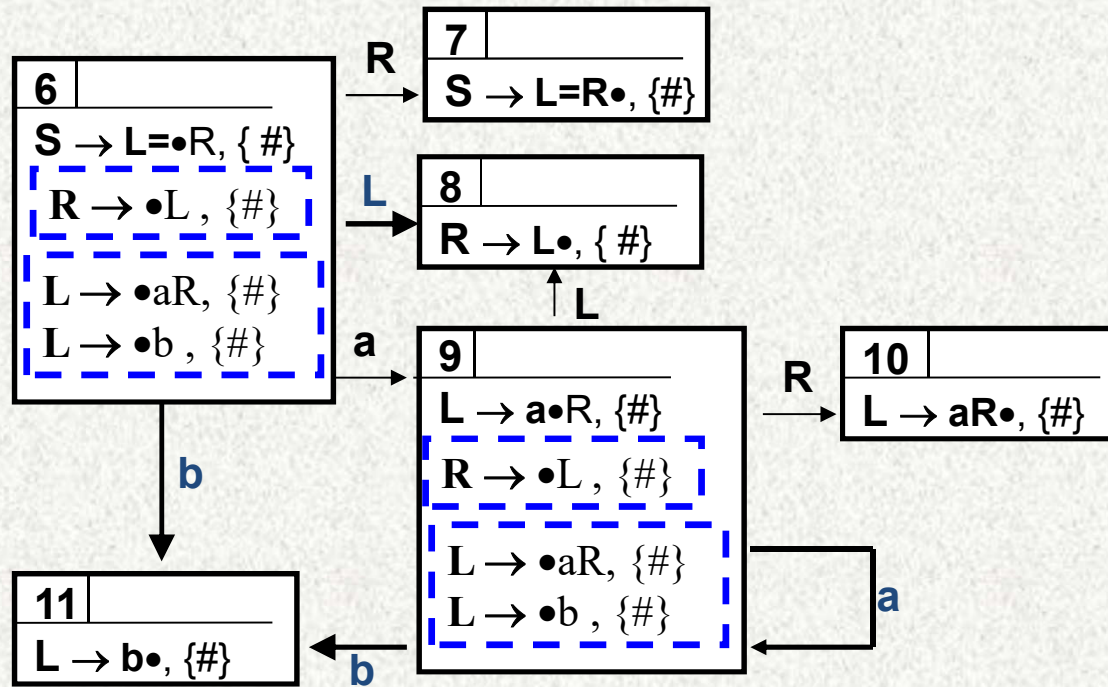


Example 2

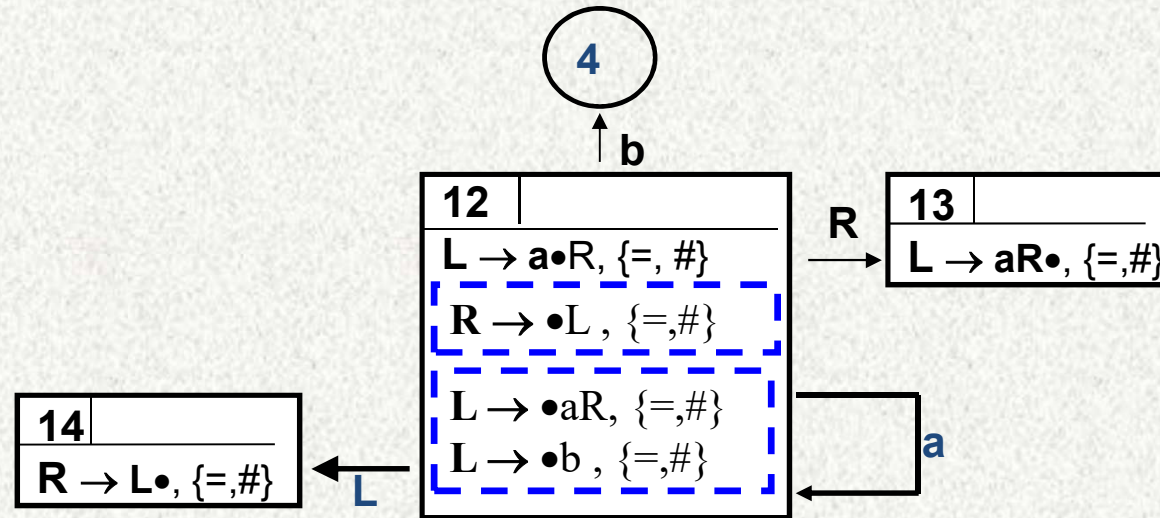
$V_T = \{a, b, =\}$
$V_N = \{S, L, R\}$
$S = S$
P: {(1) $S \rightarrow L = R$ (2) $S \rightarrow R$ (3) $L \rightarrow aR$ (4) $L \rightarrow b$ (5) $R \rightarrow L$ }



Example 2



Example 2



Canonical LR(1) Parsing Table

Algorithm 4.56: Construction of canonical-LR parsing tables.

INPUT: An augmented grammar G' .

OUTPUT: The canonical-LR parsing table functions ACTION and GOTO for G' .

METHOD:

1. Construct $C' = \{I_0, I_1, \dots, I_n\}$, the collection of sets of LR(1) items for G' .
2. State i of the parser is constructed from I_i . The parsing action for state i is determined as follows.
 - (a) If $[A \rightarrow \alpha \cdot a \beta, b]$ is in I_i and $\text{GOTO}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j .” Here a must be a terminal.
 - (b) If $[A \rightarrow \alpha \cdot, a]$ is in I_i , $A \neq S'$, then set $\text{ACTION}[i, a]$ to “reduce $A \rightarrow \alpha$.”
 - (c) If $[S' \rightarrow S \cdot, \$]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “accept.”

If any conflicting actions result from the above rules, we say the grammar is not LR(1). The algorithm fails to produce a parser in this case.

3. The goto transitions for state i are constructed for all nonterminals A using the rule: If $\text{GOTO}(I_i, A) = I_j$, then $\text{GOTO}[i, A] = j$.
4. All entries not defined by rules (2) and (3) are made “error.”
5. The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow \cdot S, \$]$.

Building the Action Table

Action Table

$\text{action}(S_i, a) = S_j$, if there is an edge from S_i to S_j labeled as a
 $\text{action}(S_i, a) = R_p$, only if S_i contains LR(1) item $(A \rightarrow \alpha \bullet, ss)$

Where $A \rightarrow \alpha$ is production P , $\exists a \in ss$;

$\text{action}(S_i, \#) = \text{accept}$, if S_i is acceptance state

$\text{action}(S_i, a) = \text{error}$, otherwise

Terminal symbols	a_1	...	#
States			
S_1			
...			
S_n			

Building the Goto Table-same as LR(0)

GOTO Table

$\text{goto}(S_i, A) = S_j$, if there is an edge from S_i to S_j labeled as A
 $\text{goto}(S_i, A) = \text{error}$, if there is no edge from S_i to S_j labeled as A

State \ non-terminal	A_1	...	#
S_1			
...			
S_n			

LR Family

- **LR Family**
 - covers wide range of grammars.
 - SLR – simple LR parser
 - LR – most general LR parser
 - LALR – intermediate LR parser (look-head LR parser)

 - SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.

Unambiguous

LL(k)

LR(k)

LL(1)

LR(1)

LALR(1)

SLR(1)

LL(0)

LR(0)

Ambiguous

