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# Lecture 2: Lexical Analysis

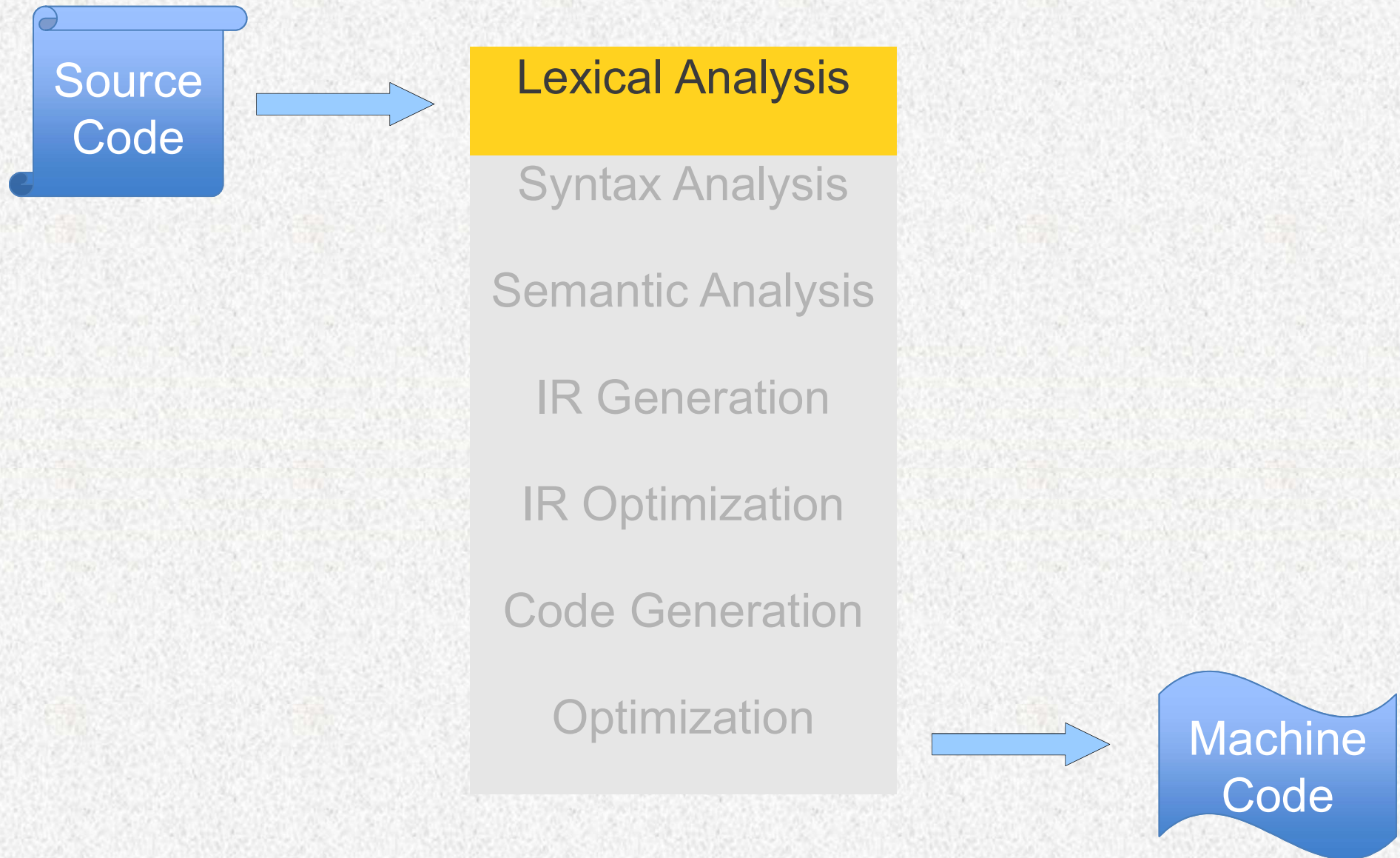
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# Where We Are







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# A motivation example

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# What do we want to do?

```
while (ip < z)
    ++ip;
```

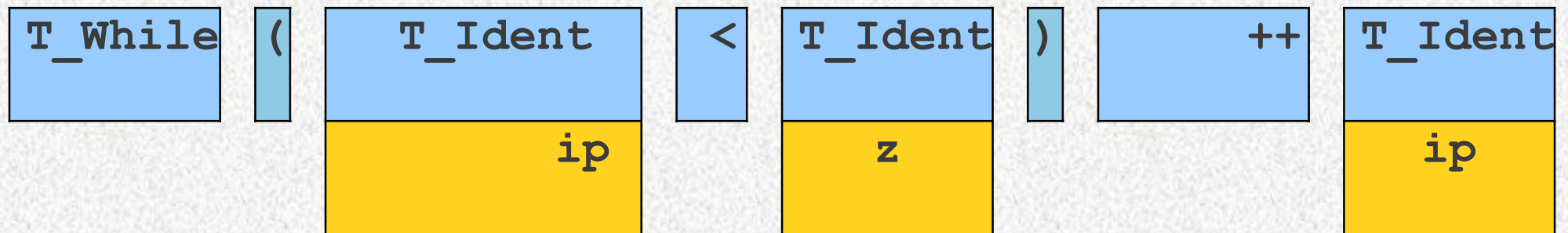


# What do we want to do?

|   |   |   |   |   |  |   |   |   |  |   |  |   |   |    |    |   |   |   |   |   |
|---|---|---|---|---|--|---|---|---|--|---|--|---|---|----|----|---|---|---|---|---|
| w | h | i | l | e |  | ( | i | p |  | < |  | z | ) | \n | \t | + | + | i | p | ; |
|---|---|---|---|---|--|---|---|---|--|---|--|---|---|----|----|---|---|---|---|---|

```
while (ip < z)
    ++ip;
```

# What do we want to do?



```
w h i l e   ( i p   <   z ) \n \t + + i p ;
```

```
while (ip < z)
    ++ip;
```



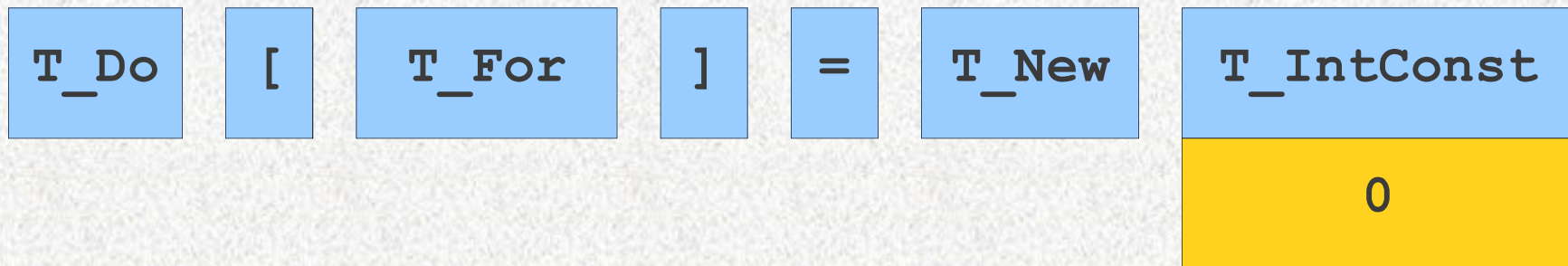
# What do we want to do?

|   |   |   |   |   |   |   |  |   |  |   |   |   |  |   |   |
|---|---|---|---|---|---|---|--|---|--|---|---|---|--|---|---|
| d | o | [ | f | o | r | ] |  | = |  | n | e | w |  | 0 | ; |
|---|---|---|---|---|---|---|--|---|--|---|---|---|--|---|---|

**do[for] = new 0;**



# What do we want to do?



d o [ f o r ] = n e w 0 ;

**do[for] = new 0;**



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|



# Scan and partition input string into substrings (i.e. tokens)

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| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|



# Scan and partition input string into substrings (i.e. tokens)

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|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
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# Scan and partition input string into substrings (i.e. tokens)

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# Scan and partition input string into substrings (i.e. tokens)

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|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

## This is straightforward

The piece of the original program from which we made the token is called a **lexeme**.

**T\_While**

This is called a **token**. You can think of it as an enumerated type representing what logical entity we read out of the source code.

## How to decide the type?

# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

**T\_While**



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

`T_While`



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

T\_While



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

**T\_While**

Sometimes we will discard a lexeme rather than storing it for later use. Here, we ignore **whitespace**, since it has no bearing on the meaning of the program.

# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

`T_While`



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

T\_While



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

T\_While



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

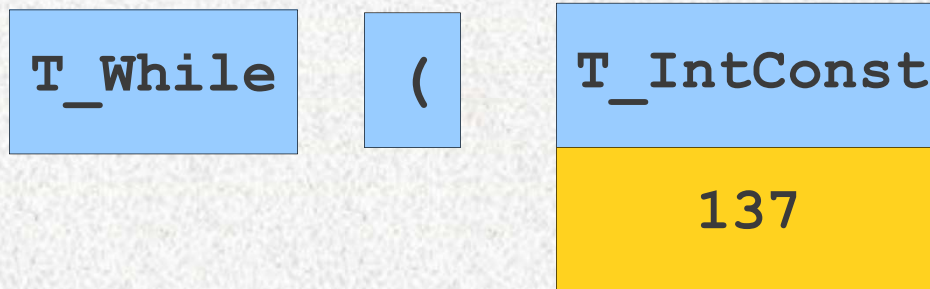
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# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|



# Scan and partition input string into substrings (i.e. tokens)

|   |   |   |   |   |  |   |   |   |   |  |   |  |   |   |    |    |   |   |   |   |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|
| w | h | i | l | e |  | ( | 1 | 3 | 7 |  | < |  | i | ) | \n | \t | + | + | i | ; |
|---|---|---|---|---|--|---|---|---|---|--|---|--|---|---|----|----|---|---|---|---|

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Some tokens can have **attributes** that store extra information about the token. Here we store which integer is represented.

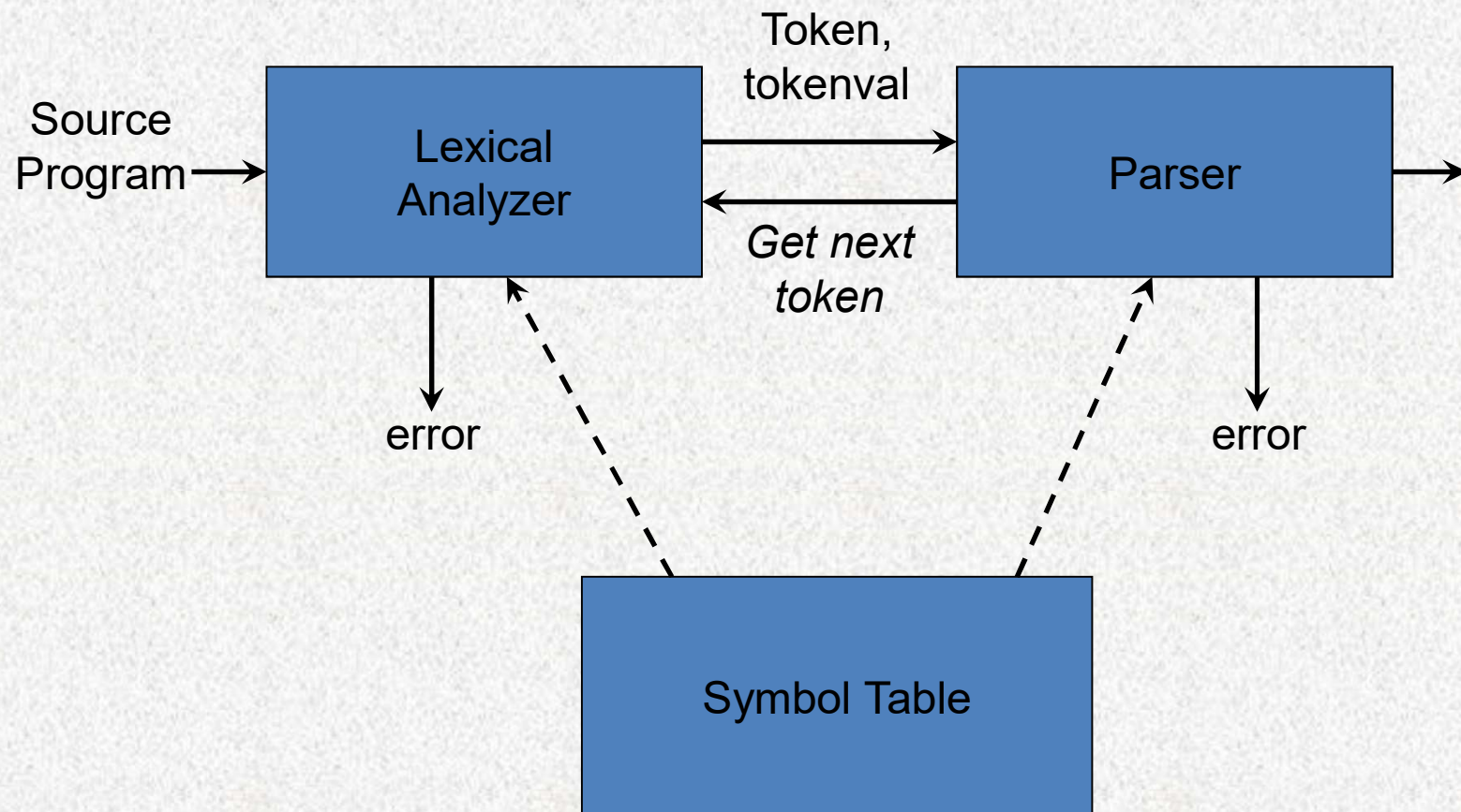


# Goals of Lexical Analysis

- Convert from physical description of a program into sequence of **tokens**.
  - Each token represents one logical piece of the source file – a keyword, the name of a variable, etc.
- Each token is associated with a **lexeme**.
- Each token may have optional **attributes**.
- Extra information derived from the text – perhaps a numeric value.
- The token sequence will be used in the parser to recover the program structure.



# Interaction of the Lexical Analyzer with the Parser







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# What is a token

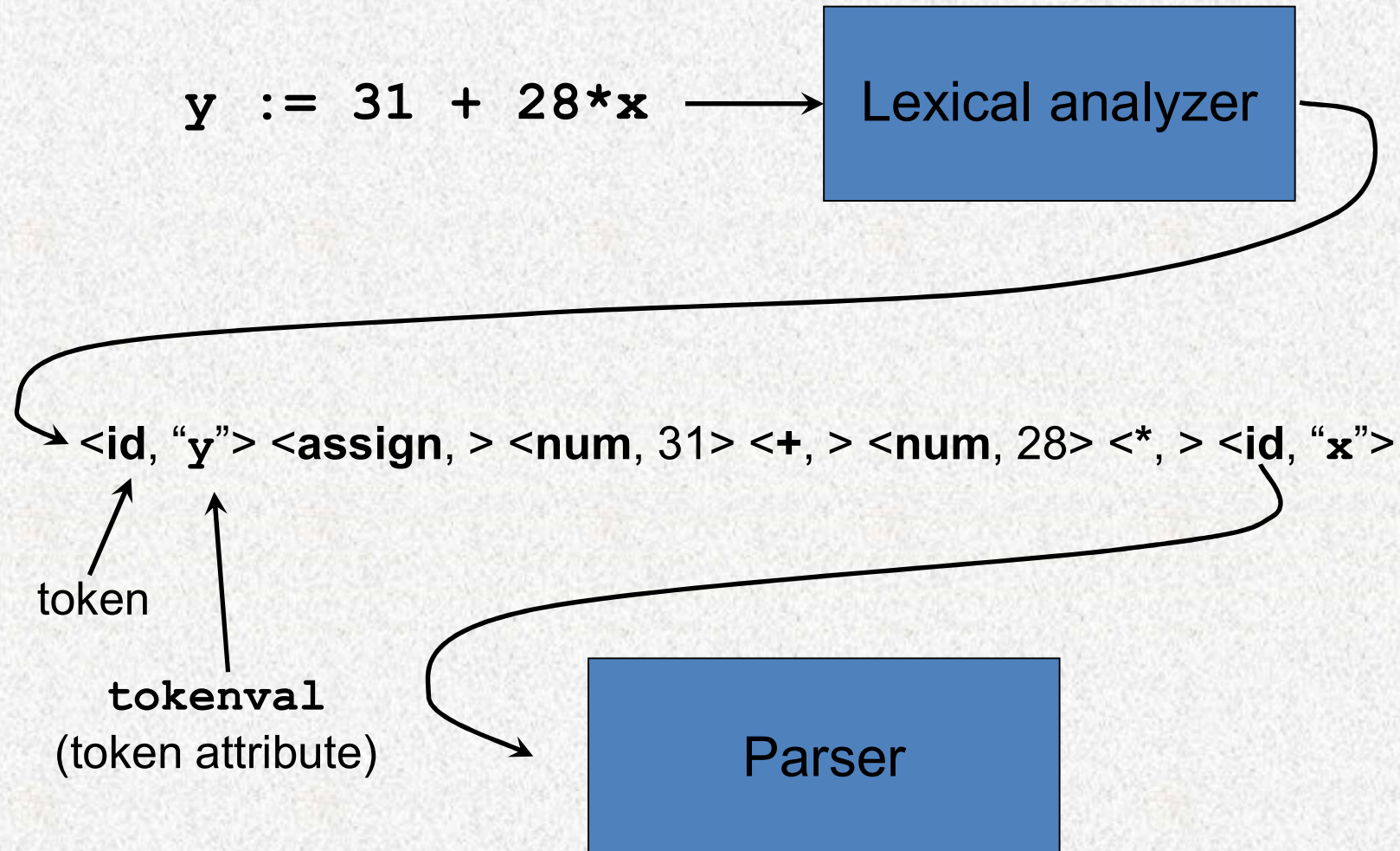
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# What is a token?

- A token should indicate a syntactic category of a lexeme
  - In English: noun, verb, adjective, ...
  - In a programming language: identifier, Integer, Keyword, Whitespace, ...



# Attributes of tokens





# What is a token?

- A token corresponds to sets of strings (a type/category/class)
  - Identifier: strings of letters or digits, starting with a letter
  - Integer: a non-empty string of digits
  - Keyword: “else” or “if” or “begin” or ...
  - Whitespace: a non-empty sequence of blanks, newlines, and tabs



# What are tokens for?

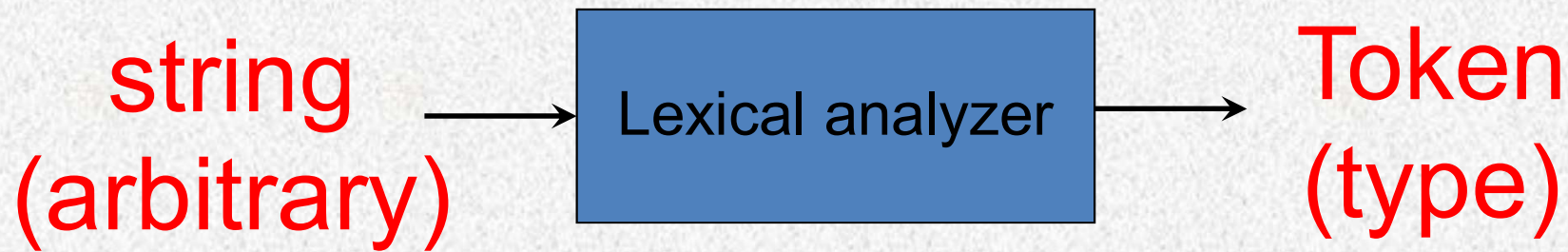
- Classify program substrings according to their roles
- Output of lexical analysis is a stream of tokens
- Parser relies on token distinctions
  - E.g. an identifier is treated differently from a keyword



# Lexemes and Tokens

- Tokens give a way to categorize lexemes by what information they provide.
- Some tokens might be associated with only a single lexeme:
  - Tokens for keywords like **if** and **while** probably only match those lexemes exactly.
- Some tokens might be associated with lots of different lexemes
  - All variable names, all possible numbers, all possible strings, etc.





**Strings are infinite**

**We need a method to describe the  
infinite strings with finite rules**



# Describe infinite strings with finite rules

- First, we define finite categories/types of tokens
  - Keywords, number, identifier, operator, etc.
- Secondly, we use finite rules to describe each type of token

How?





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# Formalisms of tokens

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# Regular languages

- Regular languages are used to define the category/type of a token in finite rules
- Three ways to describe a regular language
  - Grammar, Regular Expression, Finite Automaton
  - Equivalent to each other

Any grammar can be regarded as a generating device: derive infinite set of strings (i.e. language)



# Formally define Languages

- An *alphabet*  $\Sigma$  is a finite set of symbols (characters)
- A *string*  $s$  is a finite sequence of symbols from  $\Sigma$ 
  - $|s|$  denotes the length of string  $s$
  - $\varepsilon$  denotes the empty string, thus  $|\varepsilon| = 0$
- A *language* is a specific set of strings over some fixed alphabet  $\Sigma$  (a subset of all possible strings)



# Examples of languages

Type-III:

Alphabet = English characters

Language = English words

*Not every string of English characters is an English word!*

Type-II:

Alphabet = English characters

Language = English sentences

*Not every string of English characters is an English word!*



# Examples of languages

Type-III:

Alphabet = ASCII

Language = C tokens

*Not every string of ASCII characters is a C token!*

Type-II:

Alphabet = ASCII

Language = C programs

*Not every string of ASCII characters is a C program!*



# Examples of languages

Alphabet = English characters

Language = English words

*Not every string of English characters is an English word!*

Alphabet = ASCII

Language = C programs

Regular language is (Type-III) language

--- regular expression

--- finite automaton



# Regular Expression

# Finite Automaton



# Regular Expression

## Finite Automaton



# Regular Expressions

- **Regular expressions** are a family of descriptions that can be used to capture certain languages (i.e. the *regular languages*).
- Often provide a compact and human-readable description of the language.
- Used as the basis for numerous software systems, e.g. `flex`, `antlr`.

Identifier: strings of letters or digits, starting with a letter

letter = 'A' | ... | 'Z' | 'a' | ... | 'z'

identifier = letter (letter | digit)\*



# Atomic Regular Expressions

- The regular expressions we will use in this course begin with two simple building blocks.
  - The symbol  $\epsilon$  is a regular expression matches the empty string.
  - For any symbol  $a$ , the symbol  $a$  is a regular expression that just matches  $a$ .



# Compound Regular Expressions

1. If  $R_1$  and  $R_2$  are regular expressions,  $R_1R_2$  is a regular expression represents the **concatenation** of the languages of  $R_1$  and  $R_2$ .
2. If  $R_1$  and  $R_2$  are regular expressions,  $R_1 | R_2$  is a regular expression representing the **union** of  $R_1$  and  $R_2$ .
3. If  $R$  is a regular expression,  $R^*$  is a regular expression for the **Kleene closure** of  $R$ , **that is to repeat  $R$  for 0-n times**
4. If  $R$  is a regular expression,  $(R)$  is a regular expression with the same meaning as  $R$ .



# Operator Precedence

- Regular expression operator precedence is

$(R)$

$R^*$

$R_1R_2$

$R_1 | R_2$

- So  **$ab^*c|d$**  is parsed as  **$((a(b^*))c)|d$**



# Algebraic Laws for Regular Expression

| LAW                              | DESCRIPTION                                  |
|----------------------------------|--|
| $r s = s r$                      | $ $ is commutative                           |
| $r (s t) = (r s) t$              | $ $ is associate                             |
| $r(st) = (rs)t$                  | Concatenation is associate                   |
| $r(s t) = rs rt; (s t)r = sr tr$ | Concatenation distributes over $ $           |
| $\epsilon r = r\epsilon = r$     | $\epsilon$ is the identity for concatenation |
| $r^* = (r \epsilon)^*$           | $\epsilon$ is guaranteed in a closure        |
| $r^{**} = r^*$                   | $*$ is idempotent                            |



# Regular Expression v.s. Regular Language

- Regular expression can represent a set of strings, which form a regular language

Let  $\Sigma = \{a, b\}$

The regular expression  $a \mid b$  denotes the language  $\{a, b\}$ .

$(a|b)(a|b)$  denotes  $\{aa, ab, ba, bb\}$ , the language of all strings of length two over the alphabet.

Another regular expression for the same language is  $aa \mid ab \mid ba \mid bb$ .

$a^*$  denotes the language consisting of all strings of zero or more a's, that is,  $\{\epsilon, a, aa, aaa, \dots\}$ .



# Regular Expression v.s. Regular Language

Let  $\Sigma = \{a, b\}$

$(a \mid b)^*$  denotes the set of all strings consisting of zero or more instances of a or b, that is, all strings of a's and b's:  $\{E, a, b, aa, ab, ba, bb, aaa, \dots\}$ . Another regular expression for the same language is  $(a^* b^*)^*$ .

$a \mid a^* b$  denotes the language  $\{a, b, ab, aab, aaab, \dots\}$ , that is, the string a and all strings consisting of zero or more a's and ending in b.



# Sample Regular Expressions

- Suppose the only characters are **0** and **1**.
- Here is a regular expression for strings containing **00** as a substring:

**$(0 | 1)^*00(0 | 1)^*$**



# Sample Regular Expressions

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**(0 | 1)\*00(0 | 1)\***



# Sample Regular Expressions

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- Here is a regular expression for strings containing **00** as a substring:

**(0 | 1)\*00(0 | 1)\***

**11011100101**  
**0000**  
**11111011110011111**



# Sample Regular Expressions

- Suppose the only characters are **0** and **1**.
- Here is a regular expression for strings containing **00** as a substring:

**(0 | 1)\*00(0 | 1)\***

**11011100101**  
**0000**  
**11111011110011111**



# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings of length exactly four:



# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
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**(0|1)(0|1)(0|1)(0|1)**



# Sample Regular Expressions

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# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings of length exactly four:

**(0|1)(0|1)(0|1)(0|1)**

**0000**

**1010**

**1111**

**1000**



# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings of length exactly four:

**(0|1)(0|1)(0|1)(0|1)**

**0000**  
**1010**  
**1111**  
**1000**



# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings of length exactly four:

**(0|1){4}**

0000

1010

1111

1000



# Sample Regular Expressions

- Suppose the only characters are 0 and 1.
- Here is a regular expression for strings that contain at most one zero:

$1^*(0 \mid \epsilon)1^*$

$1^*0?1^*$

11110111

111111

0111

0



# Applied Regular Expressions

- Suppose our alphabet is  $a$ ,  $@$ , and  $.$ , where  $a$  represents “some letter.”
- A regular expression for email addresses is

$aa^* (.aa^*)^* @ aa^*.aa^*(.aa^*)^*$

$a+ (. a^+)^* @ a+ .a^+ (.a^+)^*$

$a+ (. a^+)^* @ a+ . (. a^+)^+$

[abc@whu.edu.cn](#)



# Applied Regular Expressions

- Suppose that our alphabet is all ASCII characters.
- A regular expression for even numbers is

**(+|-)?(0|1|2|3|4|5|6|7|8|9)\*(0|2|4|6|8)**

**(+|-)?[0123456789]\*[02468]**

**(+|-)?[0-9]\*[02468]**

**42**  
**+1370**  
**-3248**  
**-9999912**



# More examples

Keyword: “else” or “if” or “begin” or ...

‘else’ | ‘if’ | ‘begin’ | . . .

Integer: a non-empty string of digits

digit = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9'

integer = digit digit\*

Abbreviation:  $A^+ = AA^*$

Identifier: strings of letters or digits, starting with a letter

letter = 'A' | . . . | 'Z' | 'a' | . . . | 'z'

identifier = letter (letter | digit)\*

*Is (letter\* | digit\*) the same?*



# Regular Expression

## **Finite Automaton**



# Implementing Regular Expressions

- Regular expressions can be implemented using **finite automata**.
  - Regular expressions = **specification**
  - Finite automata = **implementation**
- There are two main kinds of finite automata:
  - **NFAs** (**nondeterministic** finite automata), which we'll see in a second, and
  - **DFAs** (**deterministic** finite automata), which we'll see later.

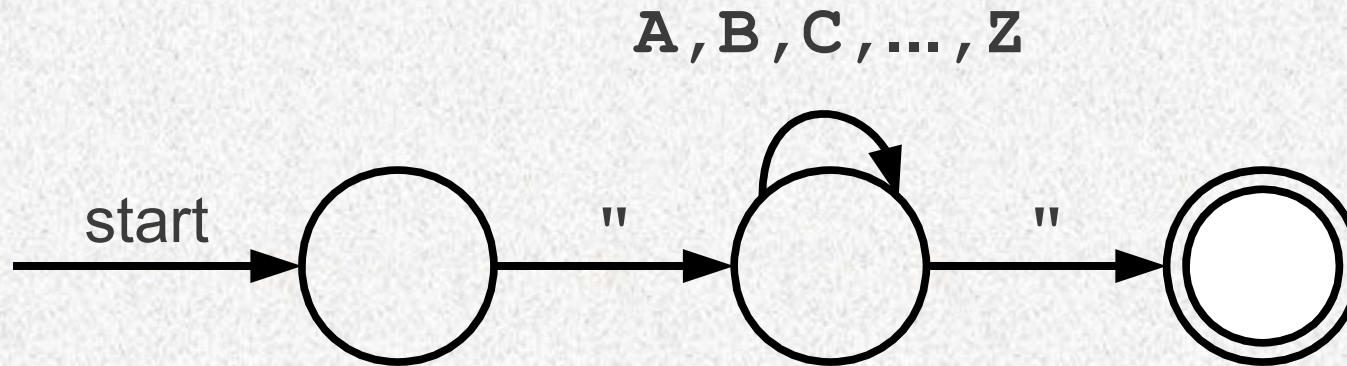


# Finite Automaton

- A finite automaton is a 5-tuple  $(S, \Sigma, \delta, s_0, F)$ 
  - A set of states  $S$  --- nodes
  - An input alphabet  $\Sigma$
  - A transition function  $\delta(S_i, a) = S_j$
  - A start state  $S_0$
  - A set of accepting states  $F \subseteq S$



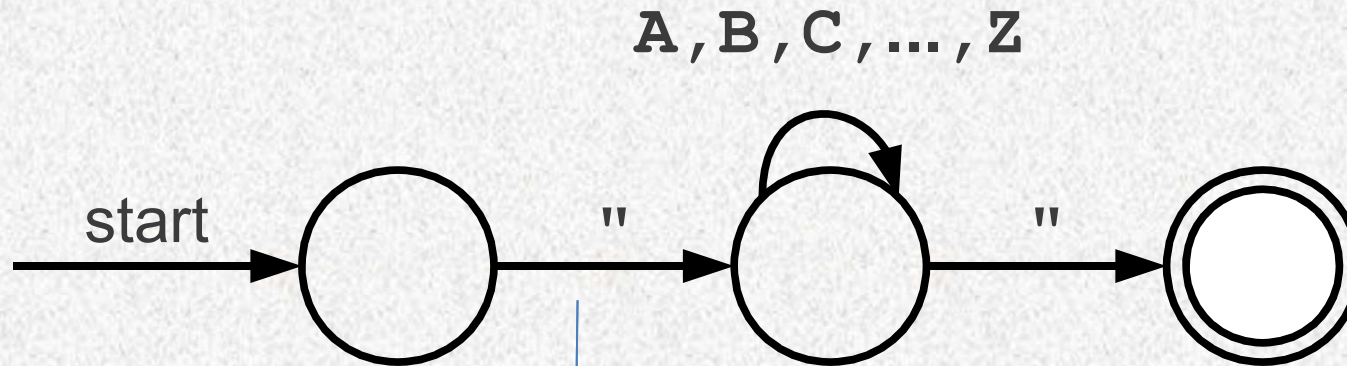
# A Simple Automaton



Transition diagrams have a collection of nodes or circles, called **states**.



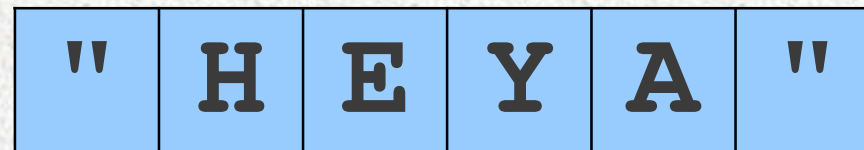
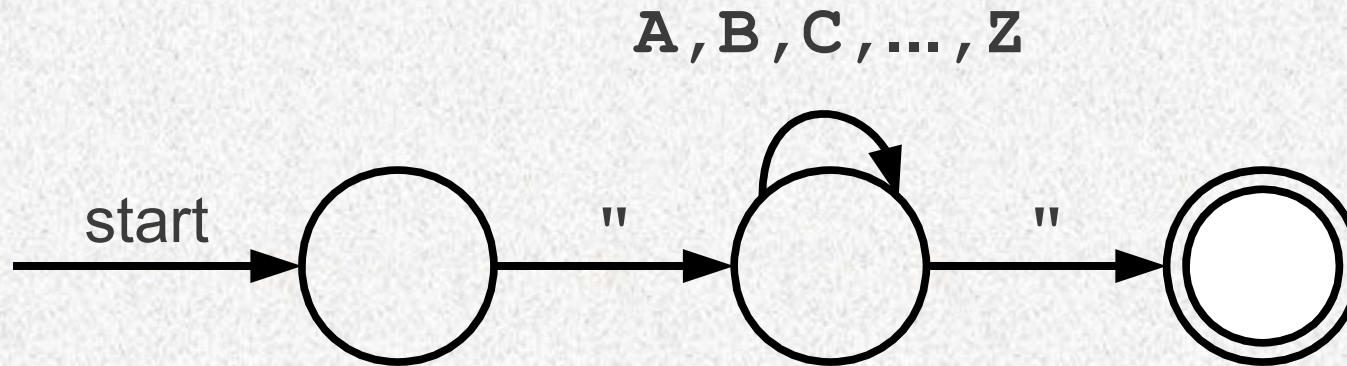
# A Simple Automaton



Arrows are called **transitions**. The automaton changes which state(s) it is in by following transitions.



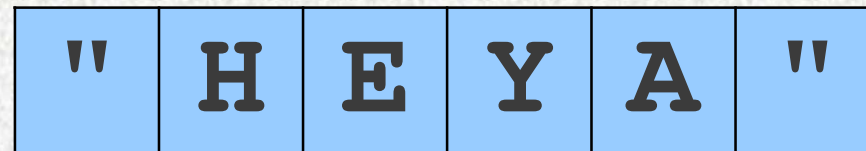
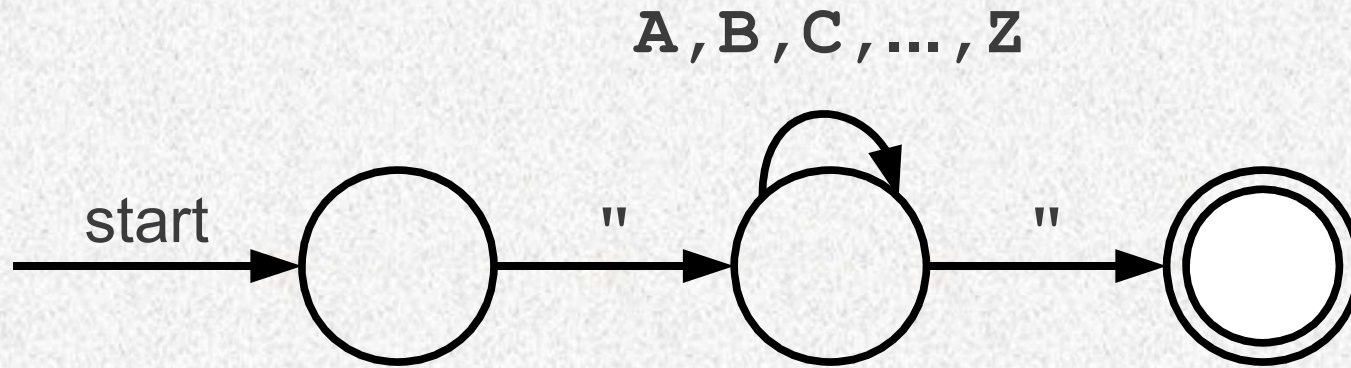
# A Simple Automaton



The automaton takes a string as input and decide whether to accept or reject the string.

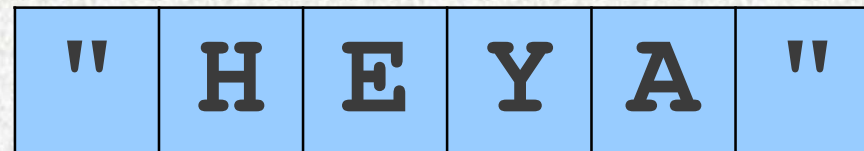
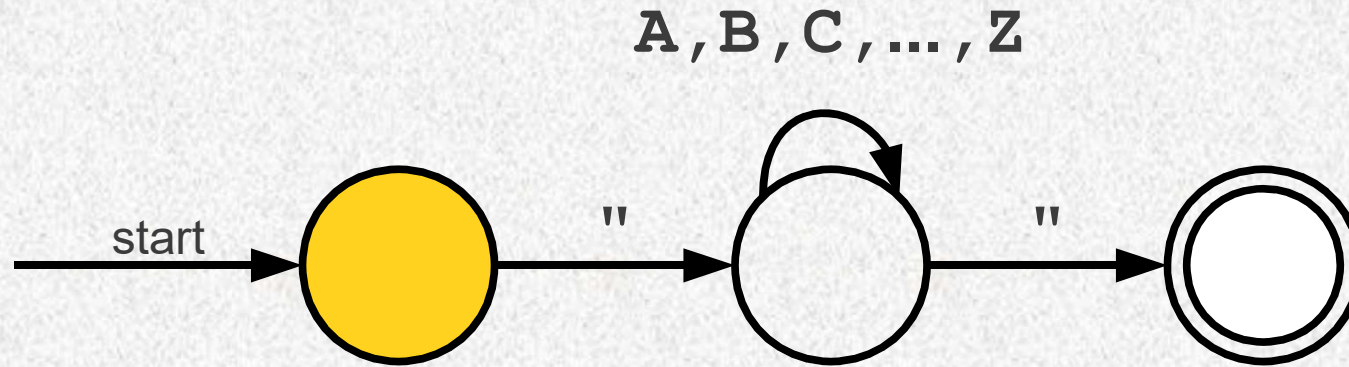


# A Simple Automaton



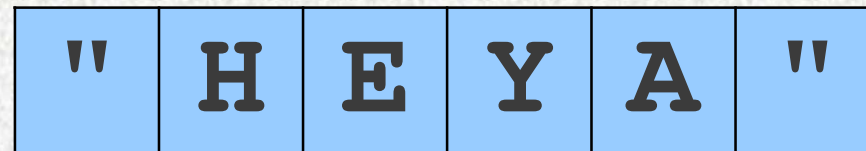
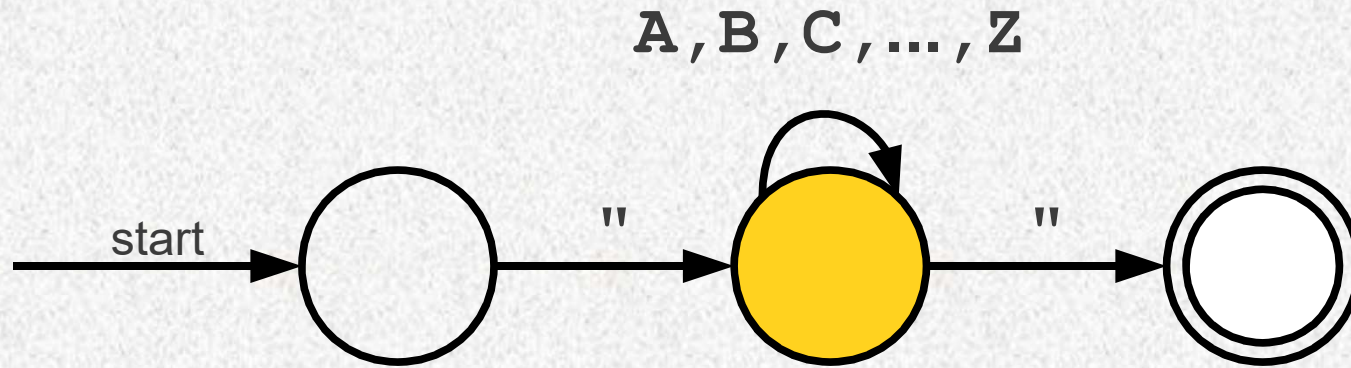


# A Simple Automaton



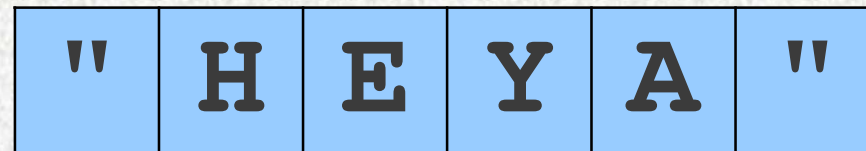
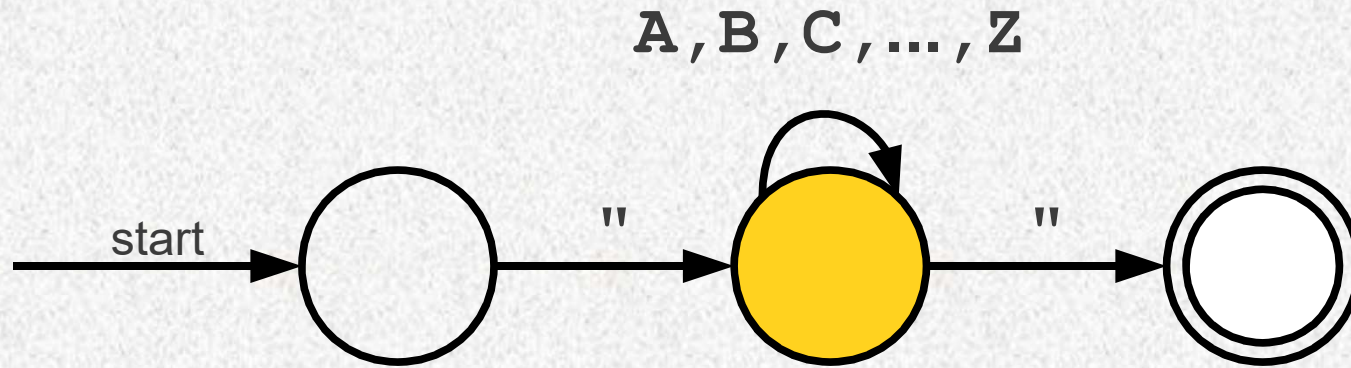


# A Simple Automaton



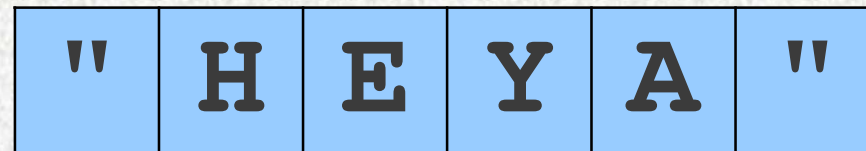
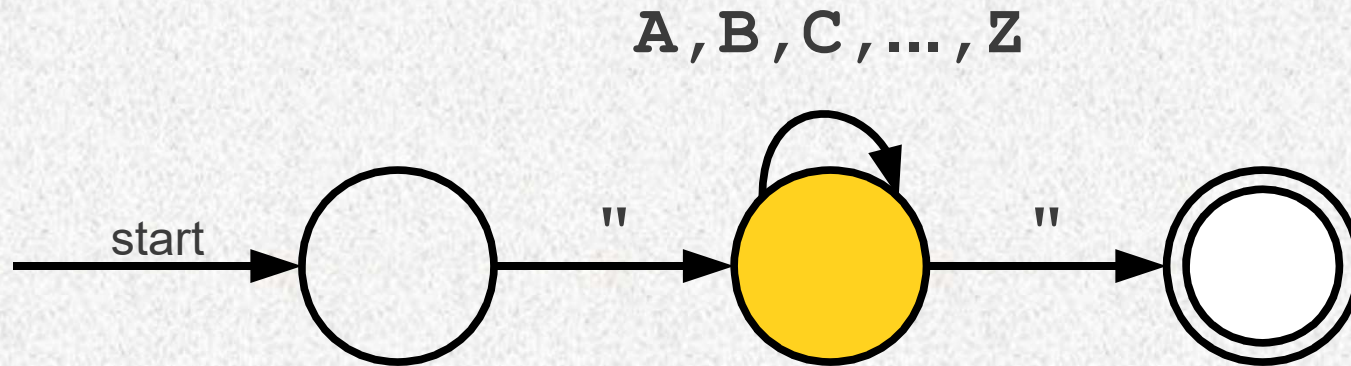


# A Simple Automaton



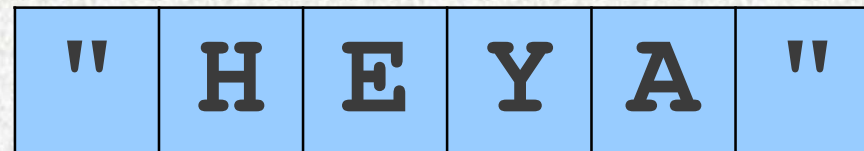
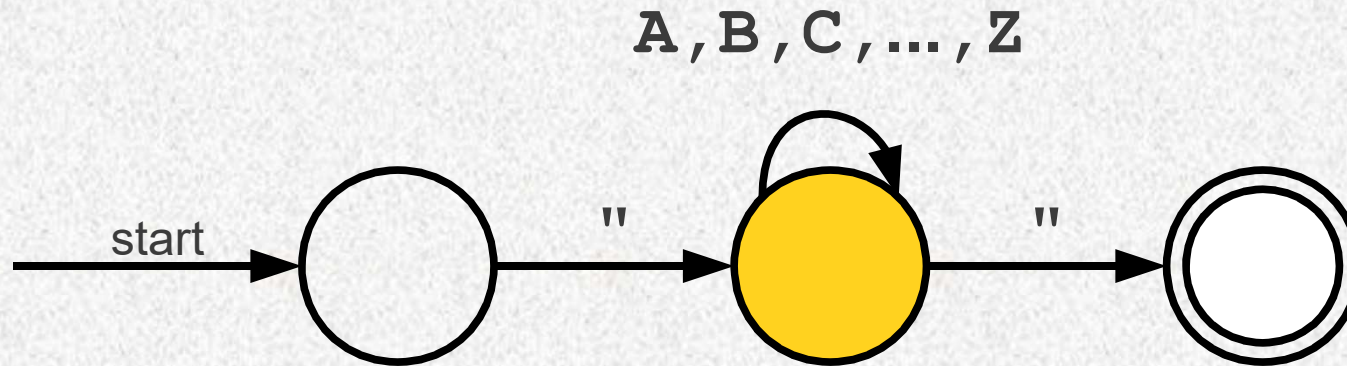


# A Simple Automaton



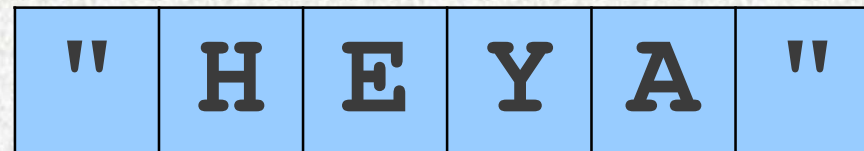
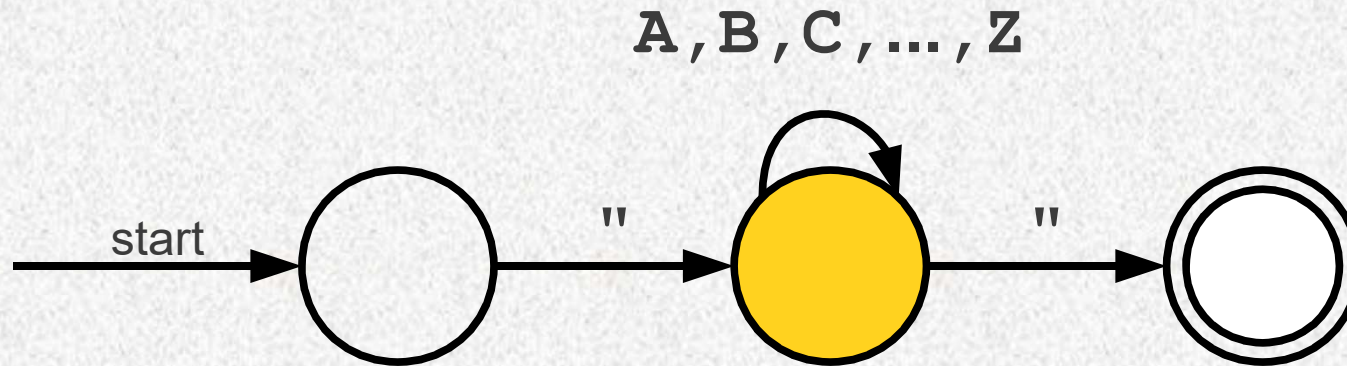


# A Simple Automaton



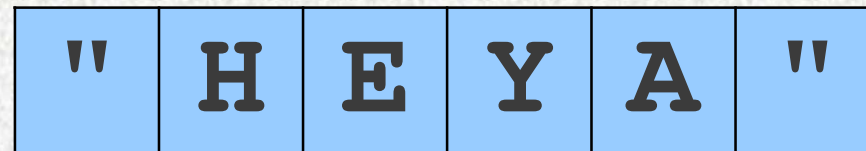
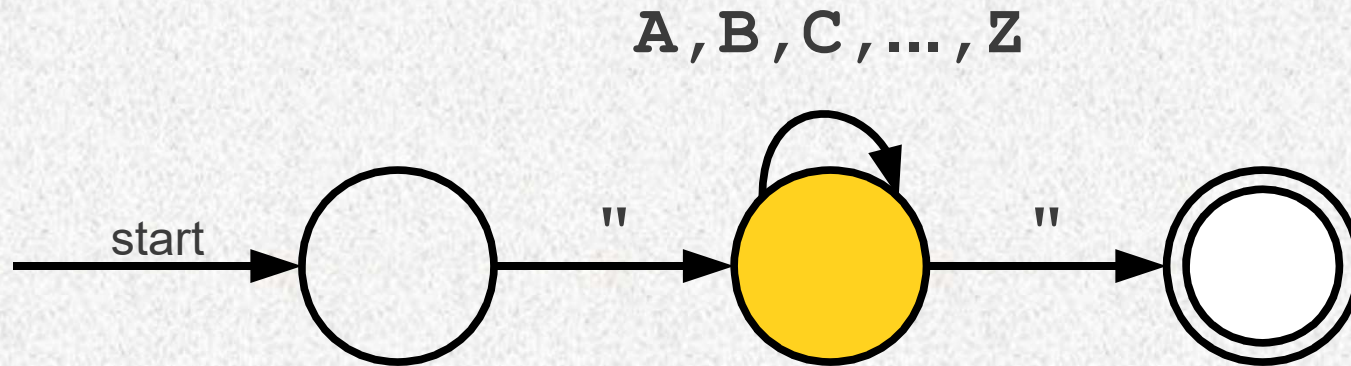


# A Simple Automaton



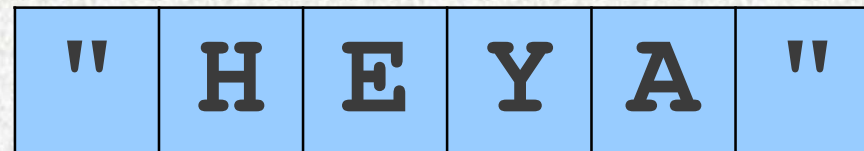
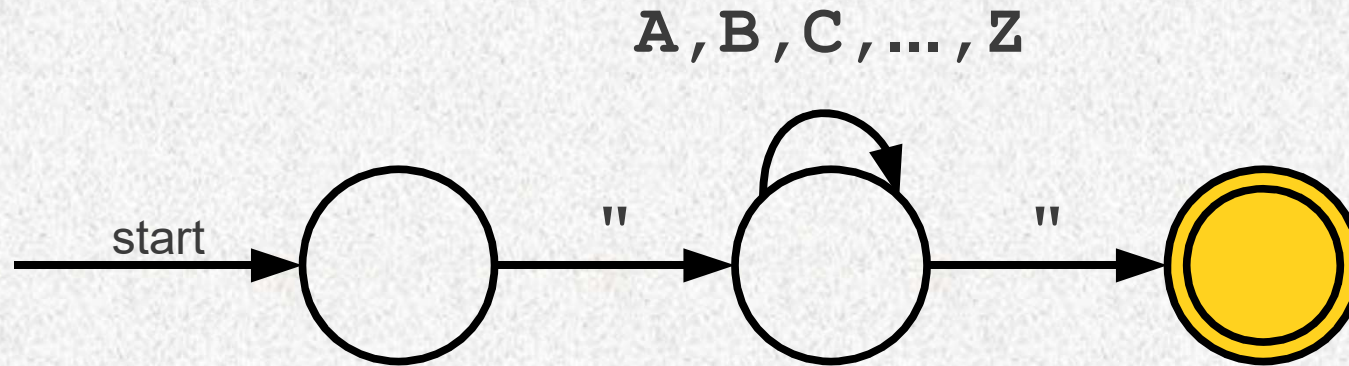


# A Simple Automaton





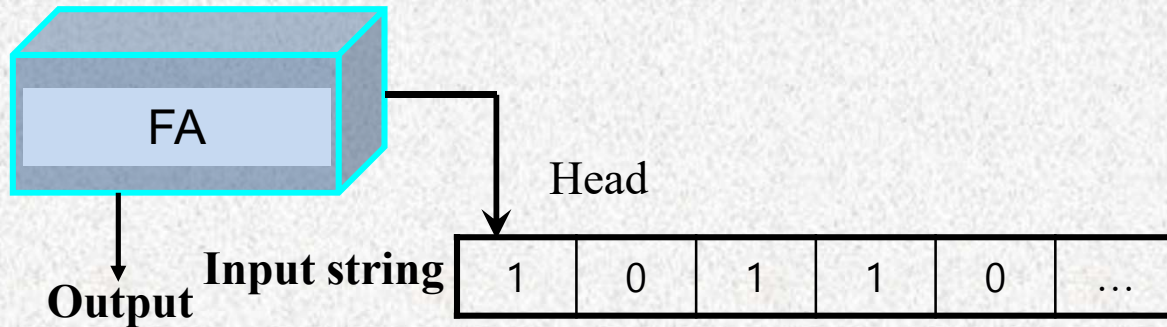
# A Simple Automaton



The double circle indicates that this state is an **accepting state**. The automaton accepts string if it ends in an accepting state.



# Finite Automaton



- Input: a string
- Output: accept if the scanning of input string reaches its EOF and the FA reaches an accepting state; reject otherwise

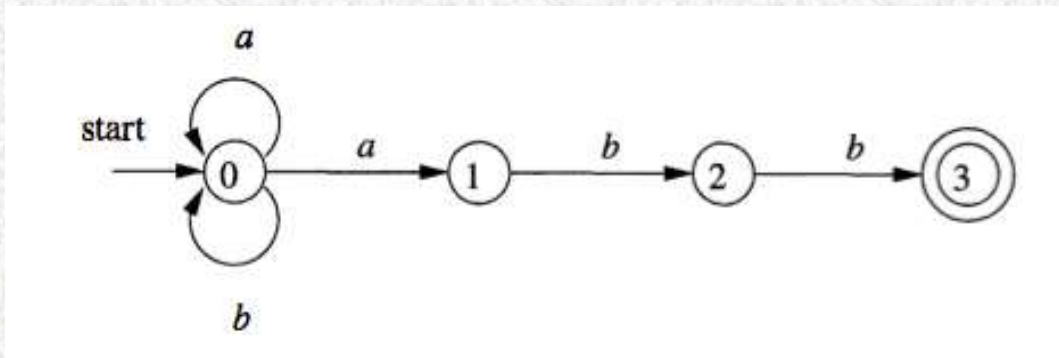


# Strings accepted by an FA

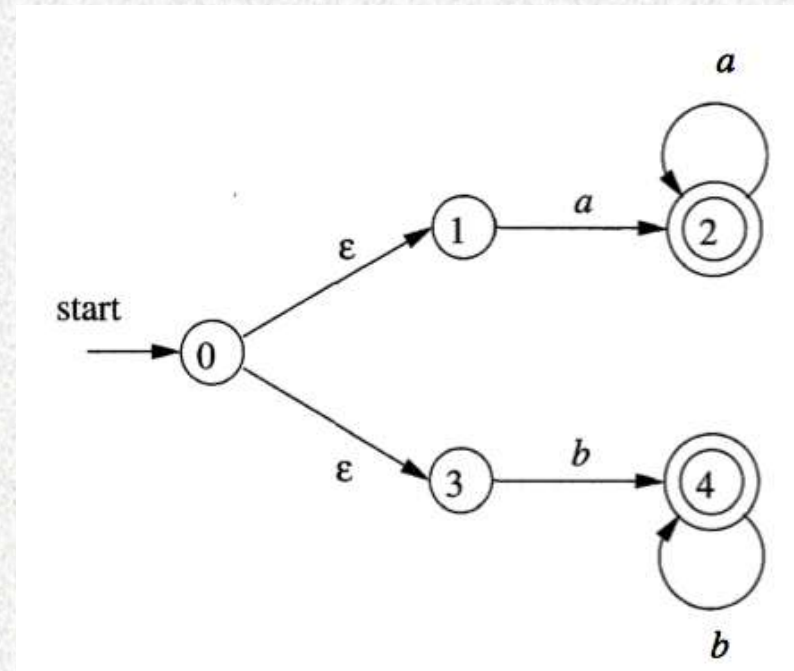
- An FA *accepts an input string  $x$*  iff there is some path with edges labeled with symbols from  $x$  in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by an FA* is *the set of input strings it accepts*, such as  $(a|b)^*abb$  for the example NFA



# Strings accepted by an FA



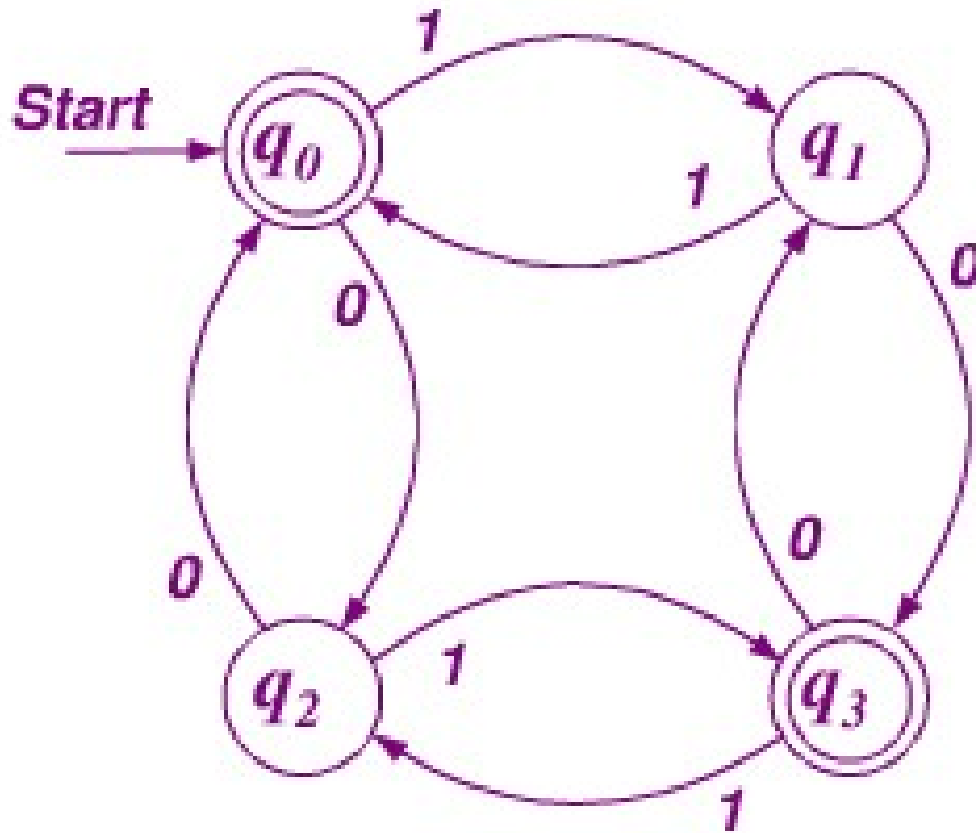
$(a|b)^*abb$



$aa^*|bb^*$



# A More Complex Automaton

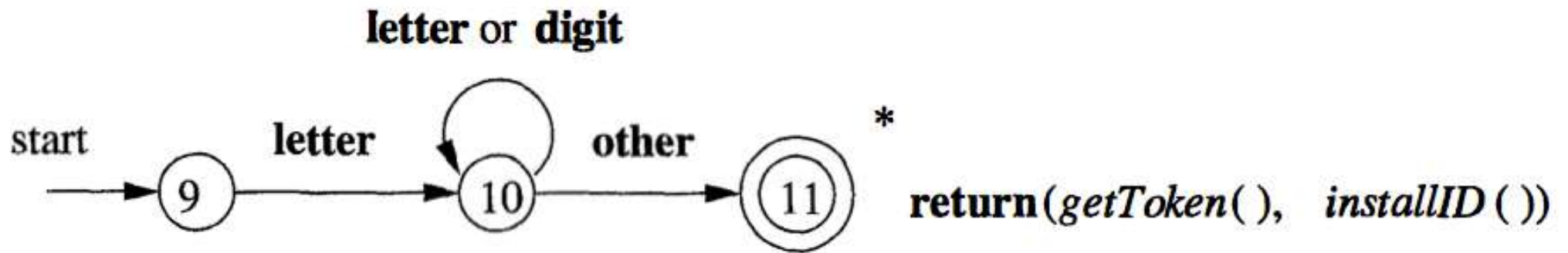


“1010”: accept

“101”: reject

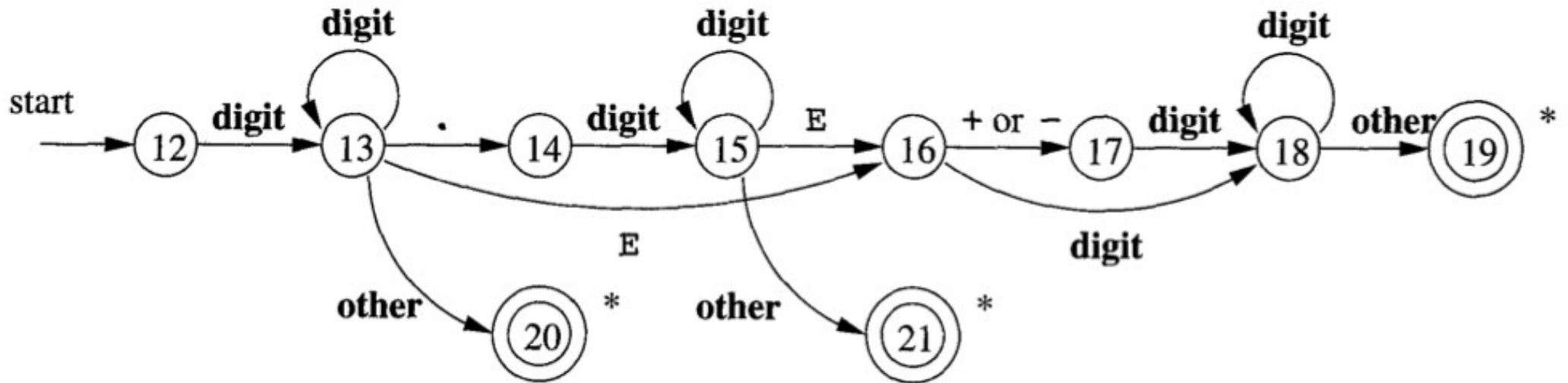


# A More Complex Automaton





# A More Complex Automaton



|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 2 | . | 3 | 7 | 5 |
|---|---|---|---|---|---|



# Finite Automata

- Finite automata is a recognizer
- Given an input string, they simply say "yes" or "no" about each possible input string



# Nondeterministic Finite Automata (NFA)

- Definition: an NFA is a 5-tuple  $(S, \Sigma, \delta, s_0, F)$  where
  - $S$  is a finite set of *states*
  - $\Sigma$  is a finite set of *input symbol alphabet*
  - $\delta$  is a *mapping* from  $S \times \Sigma \cup \{\epsilon\}$  to a set of *states*
  - $S_0 \subseteq S$  is the set of *start states*
  - $F \subseteq S$  is the set of *accepting (or final) states*



# Nondeterministic Finite Automata (NFA)

- **Transition Graph**

**Node: State**

- Non-terminal state: 
- Terminal state: 
- Starting state: 

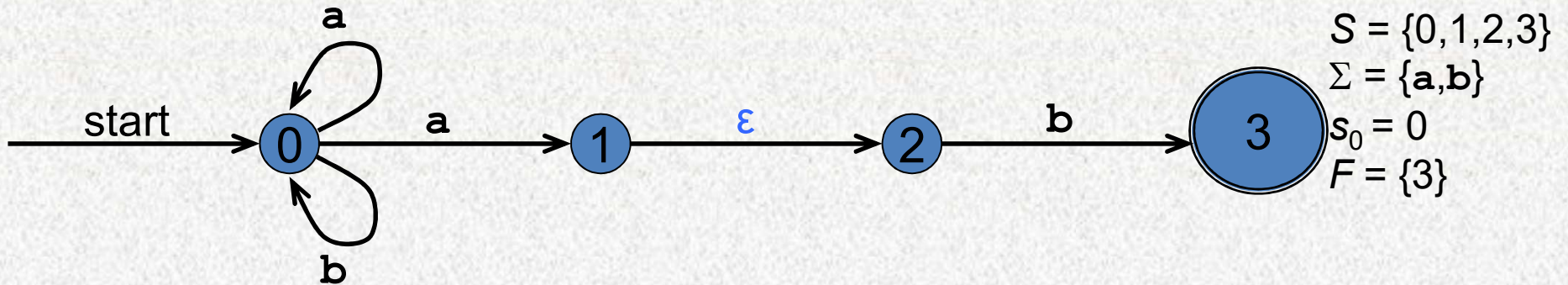
**Edge: state transition**





# Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*





# Nondeterministic Finite Automata (NFA)

- **Transit table**

- Line: State

- Starting state: in general, the first line, or label “+”;

- Terminal state: “\*” or “-”;

- Column: All symbols in  $\Sigma$

- Cell: state transition mapping



# Transition Table

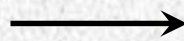
- The mapping  $\delta$  of an NFA can be represented in a *transition table*

$$\delta(0, \mathbf{a}) = \{0, 1\}$$

$$\delta(0, \mathbf{b}) = \{0\}$$

$$\delta(1, \mathbf{b}) = \{2\}$$

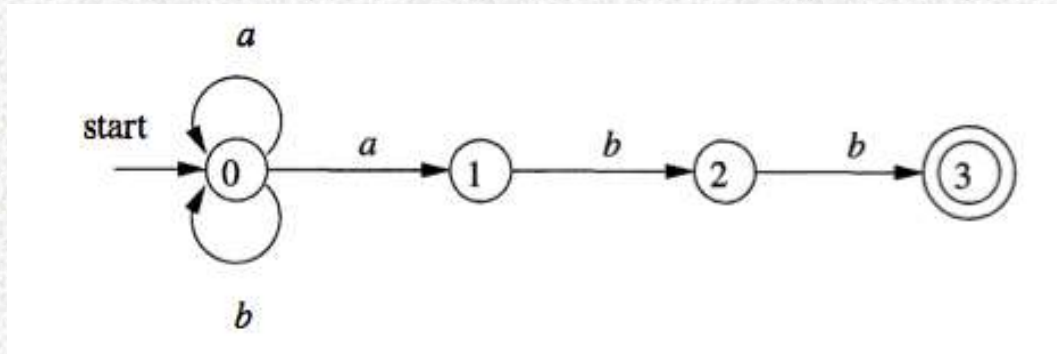
$$\delta(2, \mathbf{b}) = \{3\}$$



| <i>State</i> | <i>Input</i><br><b>a</b> | <i>Input</i><br><b>b</b> |
|--------------|--------------------------|--------------------------|
| 0            | {0,1}                    | {0}                      |
| 1            |                          | {2}                      |
| 2            |                          | {3}                      |



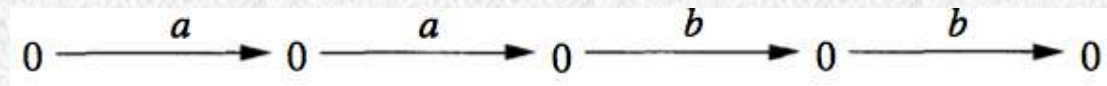
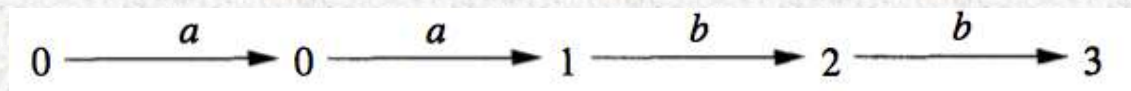
# NFA Example 2



Transition Table

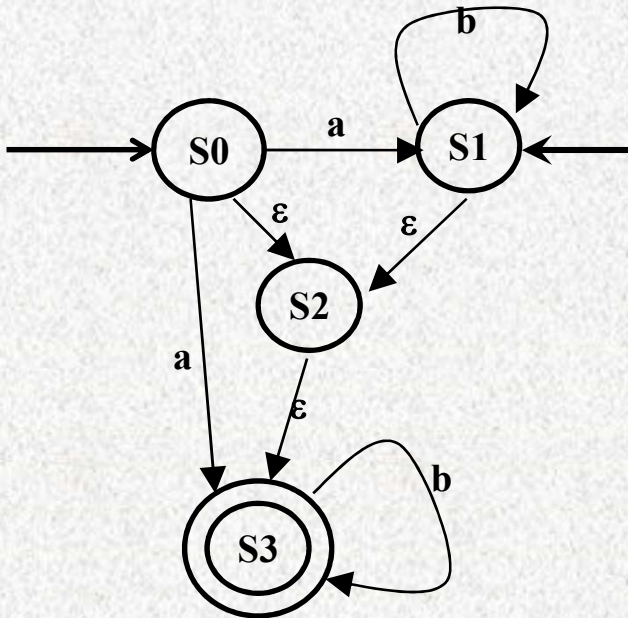
| STATE | <i>a</i>    | <i>b</i>    | $\epsilon$  |
|-------|-------------|-------------|-------------|
| 0     | {0, 1}      | {0}         | $\emptyset$ |
| 1     | $\emptyset$ | {2}         | $\emptyset$ |
| 2     | $\emptyset$ | {3}         | $\emptyset$ |
| 3     | $\emptyset$ | $\emptyset$ | $\emptyset$ |

Acceptance of input strings





# NFA Example 3



|                 | a       | b    | $\epsilon$ |
|-----------------|---------|------|------------|
| S0 <sup>+</sup> | {S1,S3} |      | {S2}       |
| S1 <sup>+</sup> |         | {S1} | {S2}       |
| S2              |         |      | {S3}       |
| S3 <sup>-</sup> |         | {S3} |            |



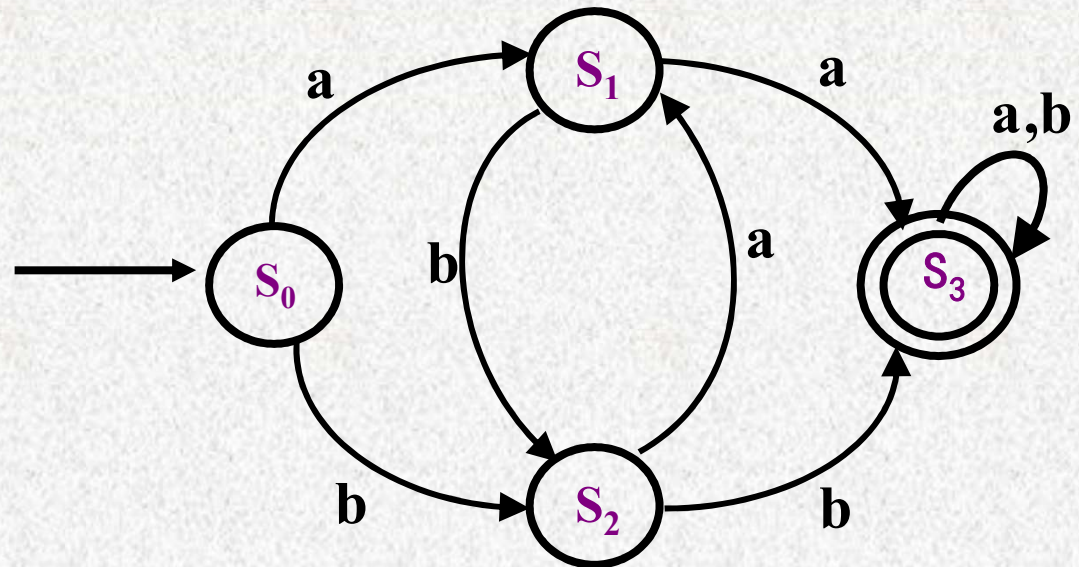
# Deterministic Finite Automata (DFA)

- Definition: an DFA is a 5-tuple  $(S, \Sigma, \delta, s_0, F)$ , is a special case of NFA
  - There are no moves on input  $\varepsilon$ , and
  - For each state  $s$  and input symbol  $a$ , there is exactly one edge out of  $s$  labeled  $a$ .



# Deterministic Finite Automata (DFA)

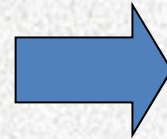
- DFA  $M = (\{S_0, S_1, S_2, S_3\}, \{a, b\}, f, S_0, \{S_3\})$ , :  
     $f(S_0, a) = S_1$                        $f(S_2, a) = S_1$   
     $f(S_0, b) = S_2$                        $f(S_2, b) = S_3$   
     $f(S_1, a) = S_3$                        $f(S_3, a) = S_3$   
     $f(S_1, b) = S_2$                        $f(S_3, b) = S_3$





# Deterministic Finite Automata (DFA)

- For example, DFA  $M = (\{0,1,2,3,4\}, \{a,b\}, \delta, \{0\}, \{3\})$
- $\delta(0, a) = 1$      $\delta(0, b) = 4$   
 $\delta(1, a) = 4$      $\delta(1, b) = 2$   
 $\delta(2, a) = 3$      $\delta(2, b) = 4$   
 $\delta(3, a) = 3$      $\delta(3, b) = 3$   
 $\delta(4, a) = 4$      $\delta(4, b) = 4$

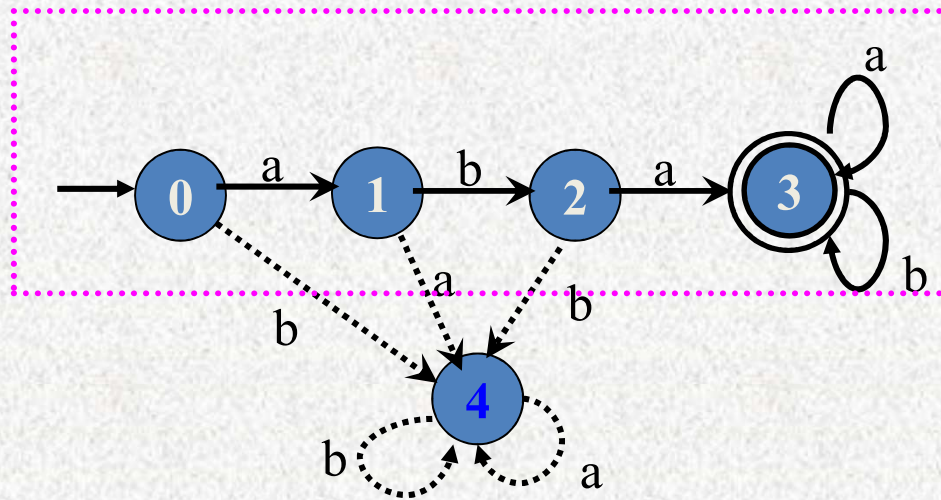


|                | a | b |
|----------------|---|---|
| 0 <sup>+</sup> | 1 | 4 |
| 1              | 4 | 2 |
| 2              | 3 | 4 |
| 3 <sup>-</sup> | 3 | 3 |
| 4              | 4 | 4 |



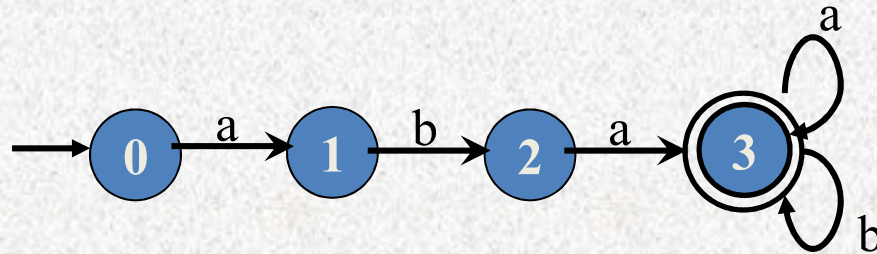
# Deterministic Finite Automata (DFA)

|    | a | b |
|----|---|---|
| 0+ | 1 | 4 |
| 1  | 4 | 2 |
| 2  | 3 | 4 |
| 3- | 3 | 3 |
| 4  | 4 | 4 |





# Deterministic Finite Automata (DFA)



|    | a | b |
|----|---|---|
| 0+ | 1 | ⊥ |
| 1  | ⊥ | 2 |
| 2  | 3 | ⊥ |
| 3- | 3 | 3 |



|    | a | b |
|----|---|---|
| 0+ | 1 |   |
| 1  |   | 2 |
| 2  | 3 |   |
| 3- | 3 | 3 |



# Deterministic Finite Automata (DFA)

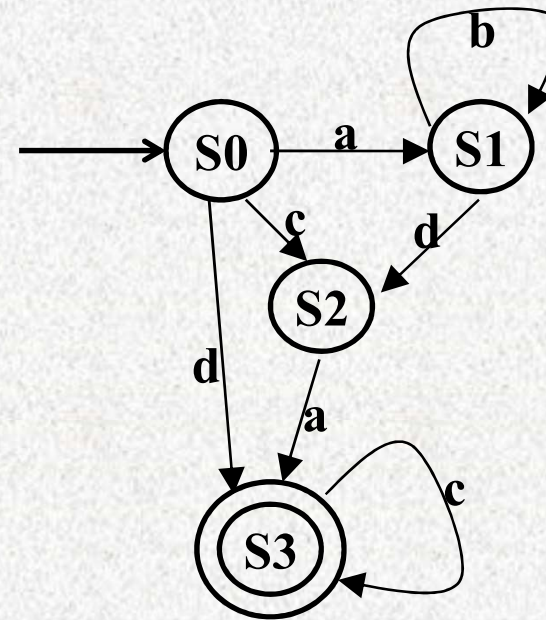
$\Sigma$ : {a, b, c, d}

S: {S0, S1, S2, S3}

Start: S0

Terminal: {S3}

f: {(S0,a)→ S1, (S0,c)→S2,  
(S0,d)→S3, (S1,b)→S1,  
(S1,d)→S2, (S2,a)→S3,  
(S3, c)→S3}





# NFA v.s. DFA



# NFA v.s. DFA


|                 | DFA                   | NFA                              |
|-----------------|-----------------------|----------------------------------|
| Initial         | Single starting state | A set of starting states         |
| $\epsilon$ dege | Not allowed           | Allowed                          |
| $\delta(S, a)$  | $S'$ or $\perp$       | $\{S_1, \dots, S_n\}$ or $\perp$ |
| Implementation  | Deterministic         | Nondeterministic                 |

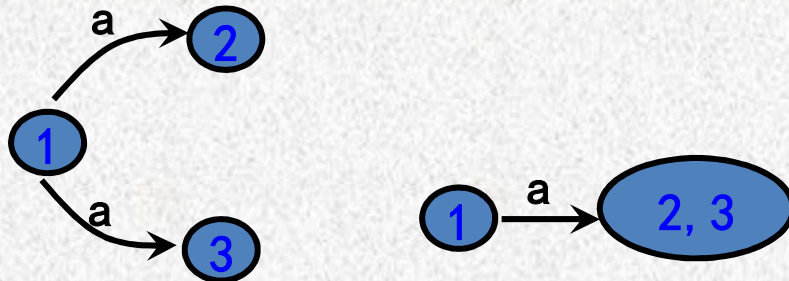
- DFA accepts an input string with only one path
- NFA accepts an input string with possibly multiple paths



# Construct DFA from NFA

- Construct DFA from NFA
  - For any NFA, **there exists an equivalent DFA**
  - Idea of construction: eliminate the uncertainty
  - Merge N states in NFA into **one single state**

- Eliminate  $\epsilon$  
  - Eliminate multiple mapping





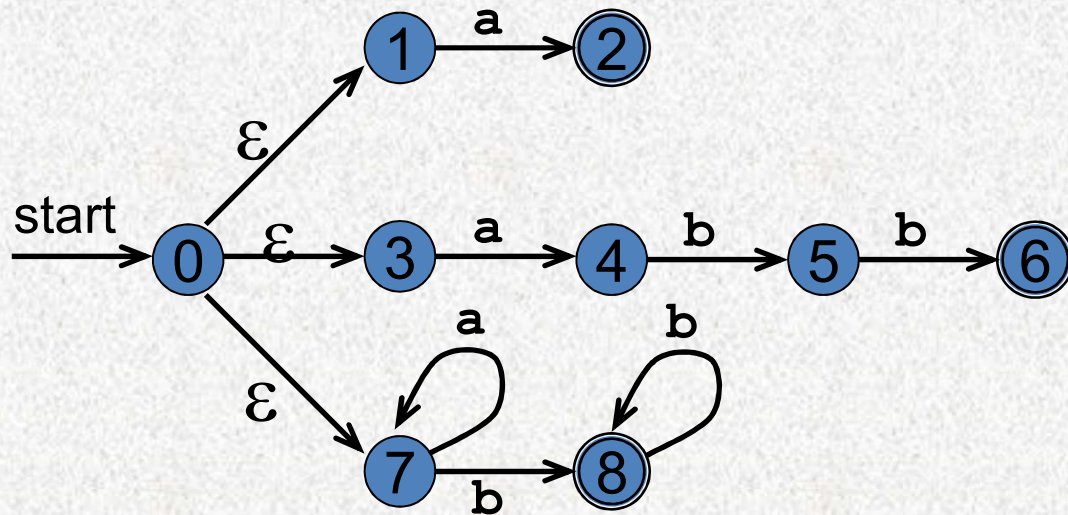
# Construct DFA from NFA

- **INPUT:** An NFA  $N$ .
- **OUTPUT:** A DFA  $D$  accepting the same language as  $N$ .
- **METHOD:** The algorithm constructs a transition table  $D_{\text{tran}}$  for  $D$ . Each state of  $D$  is a set of NFA states, and we construct  $D_{\text{tran}}$  so  $D$  will simulate “in parallel” all possible moves  $N$  can make on a given input string.

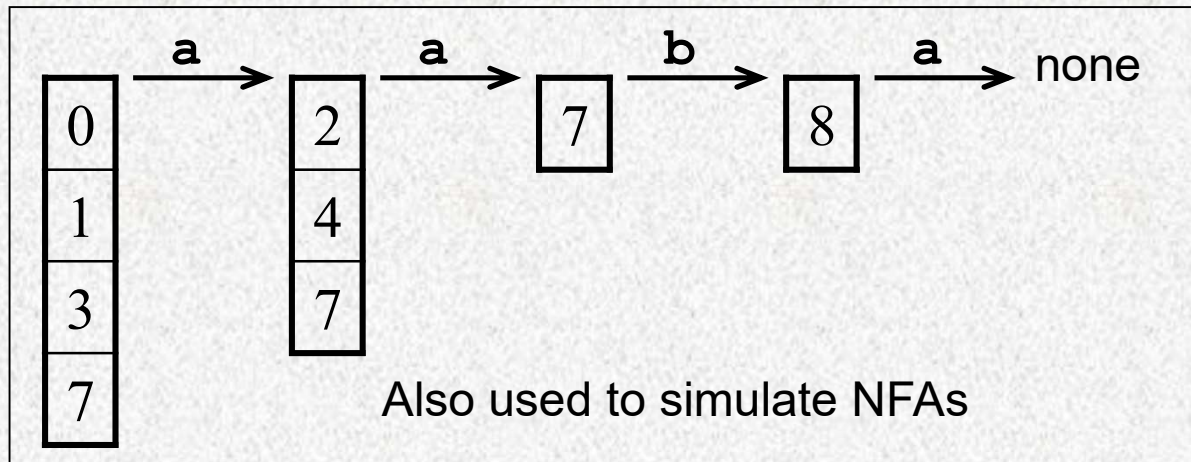
| OPERATION                    | DESCRIPTION  |
|------------------------------|--|
| $\epsilon\text{-closure}(s)$ | Set of NFA states reachable from NFA state $s$ on $\epsilon$ -transitions alone.   |
| $\epsilon\text{-closure}(T)$ | Set of NFA states reachable from some NFA state $s$ in set $T$ on $\epsilon$ -transitions alone; $= \cup_{s \text{ in } T} \epsilon\text{-closure}(s)$ . |
| $\text{move}(T, a)$          | Set of NFA states to which there is a transition on input symbol $a$ from some state $s$ in $T$ .  |



# $\epsilon$ -closure and *move* Examples



$\epsilon$ -closure( $\{0\}$ ) =  $\{0, 1, 3, 7\}$   
 $move(\{0, 1, 3, 7\}, a)$  =  $\{2, 4, 7\}$   
 $\epsilon$ -closure( $\{2, 4, 7\}$ ) =  $\{2, 4, 7\}$   
 $move(\{2, 4, 7\}, a)$  =  $\{7\}$   
 $\epsilon$ -closure( $\{7\}$ ) =  $\{7\}$   
 $move(\{7\}, b)$  =  $\{8\}$   
 $\epsilon$ -closure( $\{8\}$ ) =  $\{8\}$   
 $move(\{8\}, a)$  =  $\emptyset$





# The Subset Construction Algorithm

- NFAs can be in many states at once, while DFAs can only be in a single state at a time.
- Key idea: **Make the DFA simulate the NFA.**
- Have the states of the DFA correspond to the *sets of states* of the NFA.
- Transitions between states of DFA correspond to transitions between *sets of states* in the NFA.



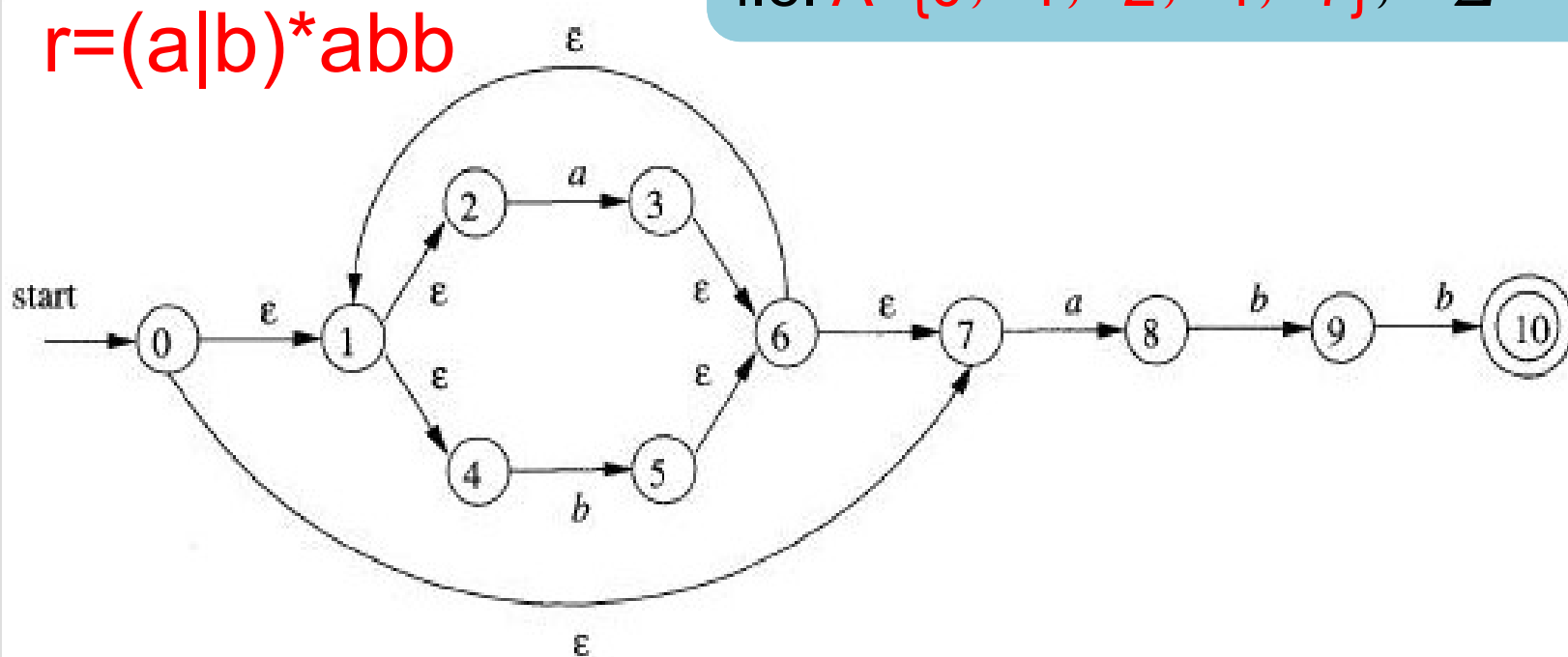
# The Subset Construction Algorithm

```
initially,  $\epsilon$ -closure( $s_0$ ) is the only state in  $Dstates$ , and it is unmarked;  
while ( there is an unmarked state  $T$  in  $Dstates$  ) {  
    mark  $T$ ;  
    for ( each input symbol  $a$  ) {  
         $U = \epsilon$ -closure(move( $T, a$ ));  
        if (  $U$  is not in  $Dstates$  )  
            add  $U$  as an unmarked state to  $Dstates$ ;  
         $Dtran[T, a] = U$ ;  
    }  
}
```



# Subset Construction Example 1

First, Initial state of NFA is  $\epsilon$ -closure(0),  
i.e.  $A = \{0, 1, 2, 4, 7\}$ ,  $\Sigma = \{a, b\}$



$Dtran[A, a] = \epsilon$ -closure(move( $A, a$ )) =  $\epsilon$ -closure( $\{3, 8\}$ ) =  $\{1, 2, 3, 4, 6, 7, 8\}$ ,

Let  $B = Dtran[A, a]$

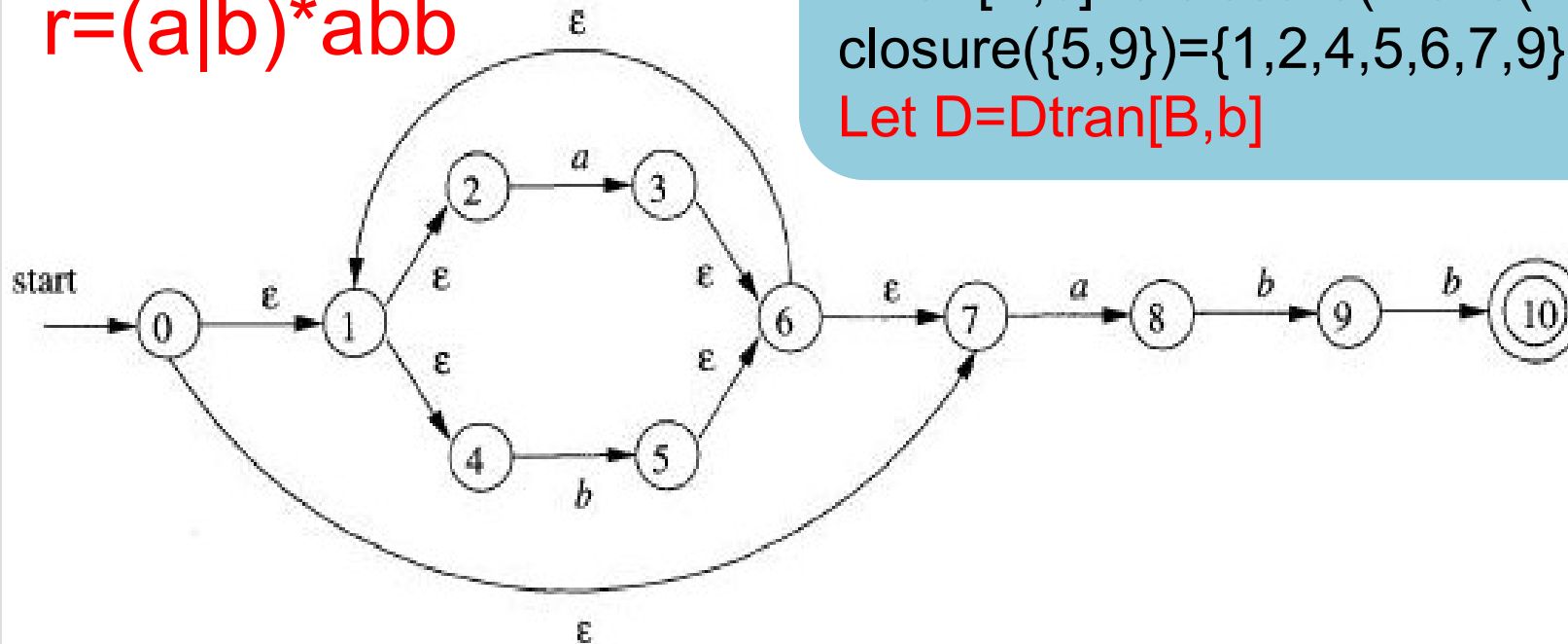
$Dtran[A, b] = \epsilon$ -closure(move( $A, b$ )) =  $\epsilon$ -closure( $\{5\}$ ) =  $\{1, 2, 4, 6, 7\}$ ,

Let  $C = Dtran[A, b]$



# Subset Construction Example 1

$r = (a|b)^*abb$



$D_{\text{tran}}[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{3, 8\}) = \{1, 2, 3, 4, 6, 7, 8\} = B$

$D_{\text{tran}}[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{5, 9\}) = \{1, 2, 4, 5, 6, 7, 9\}$ ,

Let  $D = D_{\text{tran}}[B, b]$

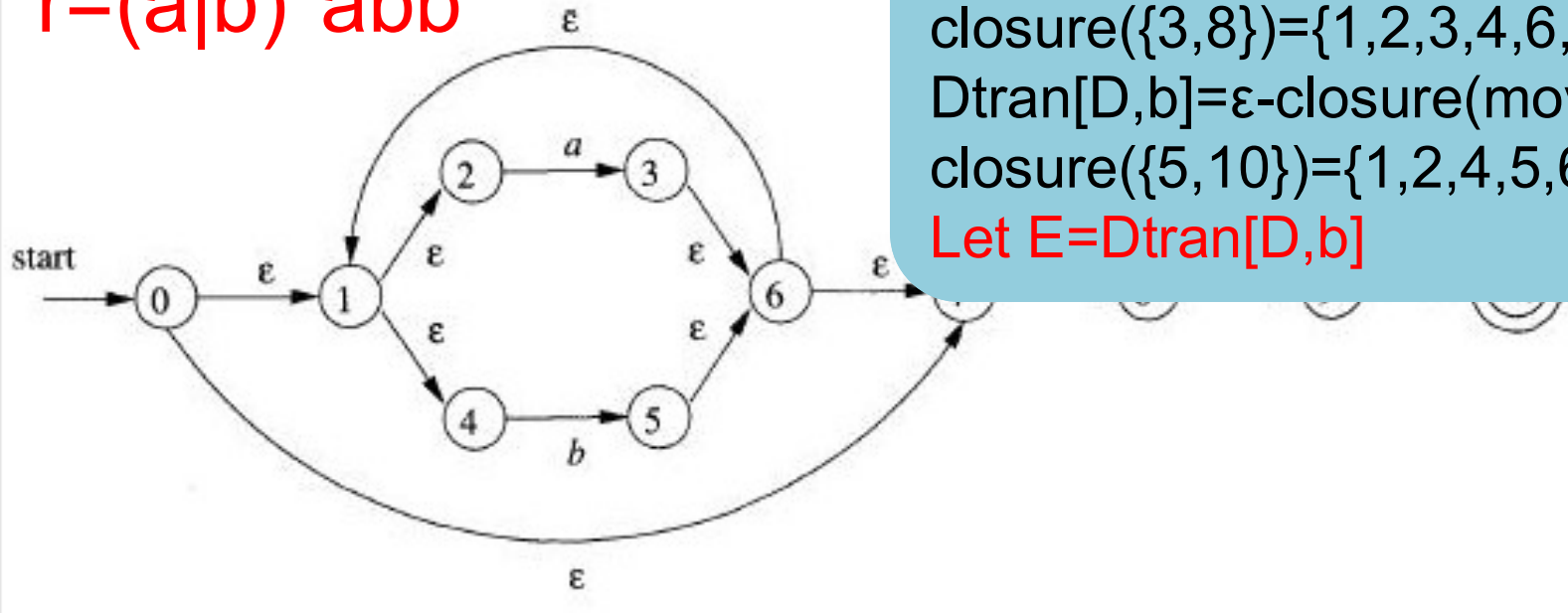
$D_{\text{tran}}[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{3, 8\}) = \{1, 2, 3, 4, 6, 7, 8\} = B$

$D_{\text{tran}}[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{5\}) = \{1, 2, 4, 6, 7\} = C$



# Subset Construction Example 1

$r = (a|b)^*abb$



$Dtran[D,a] = \epsilon\text{-closure}(\text{move}(D,a)) = \epsilon\text{-closure}(\{3,8\}) = \{1,2,3,4,6,7,8\} = B$

$Dtran[D,b] = \epsilon\text{-closure}(\text{move}(D,b)) = \epsilon\text{-closure}(\{5,10\}) = \{1,2,4,5,6,7,10\}$ ,

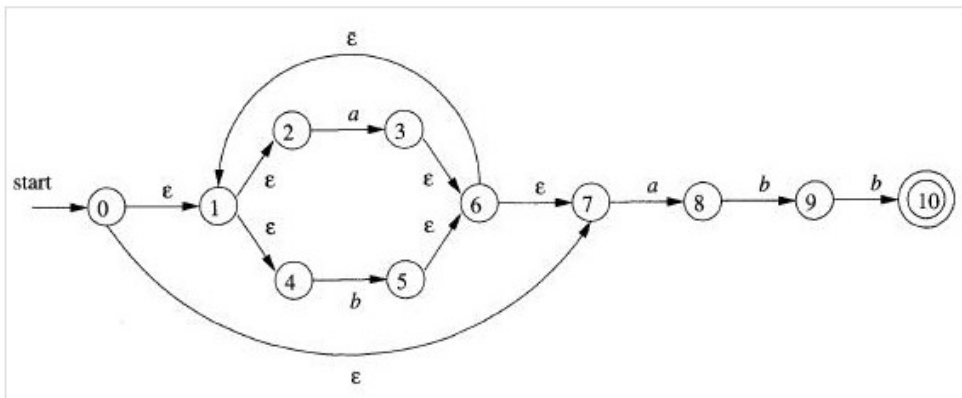
Let  $E = Dtran[D,b]$

$Dtran[E,a] = \epsilon\text{-closure}(\text{move}(E,a)) = \epsilon\text{-closure}(\{3,8\}) = \{1,2,3,4,6,7,8\} = B$

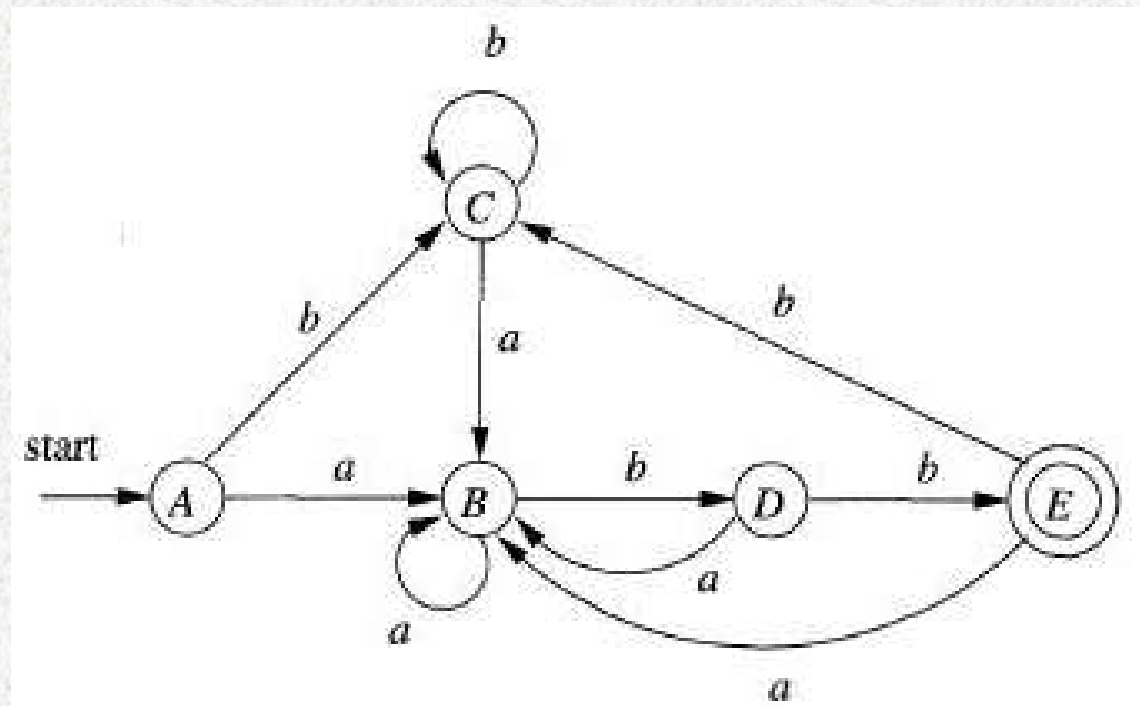
$Dtran[E,b] = \epsilon\text{-closure}(\text{move}(E,b)) = \epsilon\text{-closure}(\{5\}) = \{1,2,4,6,7\} = C$



# Subset Construction Example 1



| NFA STATE              | DFA STATE | a | b |
|------------------------|-----------|---|---|
| {0, 1, 2, 4, 7}        | A         | B | C |
| {1, 2, 3, 4, 6, 7, 8}  | B         | B | D |
| {1, 2, 4, 5, 6, 7}     | C         | B | C |
| {1, 2, 4, 5, 6, 7, 9}  | D         | B | E |
| {1, 2, 3, 5, 6, 7, 10} | E         | B | C |







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# Homework-W2

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# Homework – week 2

- pp.125, Exercise 3.3.2 (a)(c), 3.3.5 (a)(e)
- pp.151-152, Exercise 3.6.3, Exercise 3.6.4
- pp.152, Exercise 3.6.5
- pp. 166, Exercise 3.7.1 (b)