Lecture 3: Lexical Analysis Cont.

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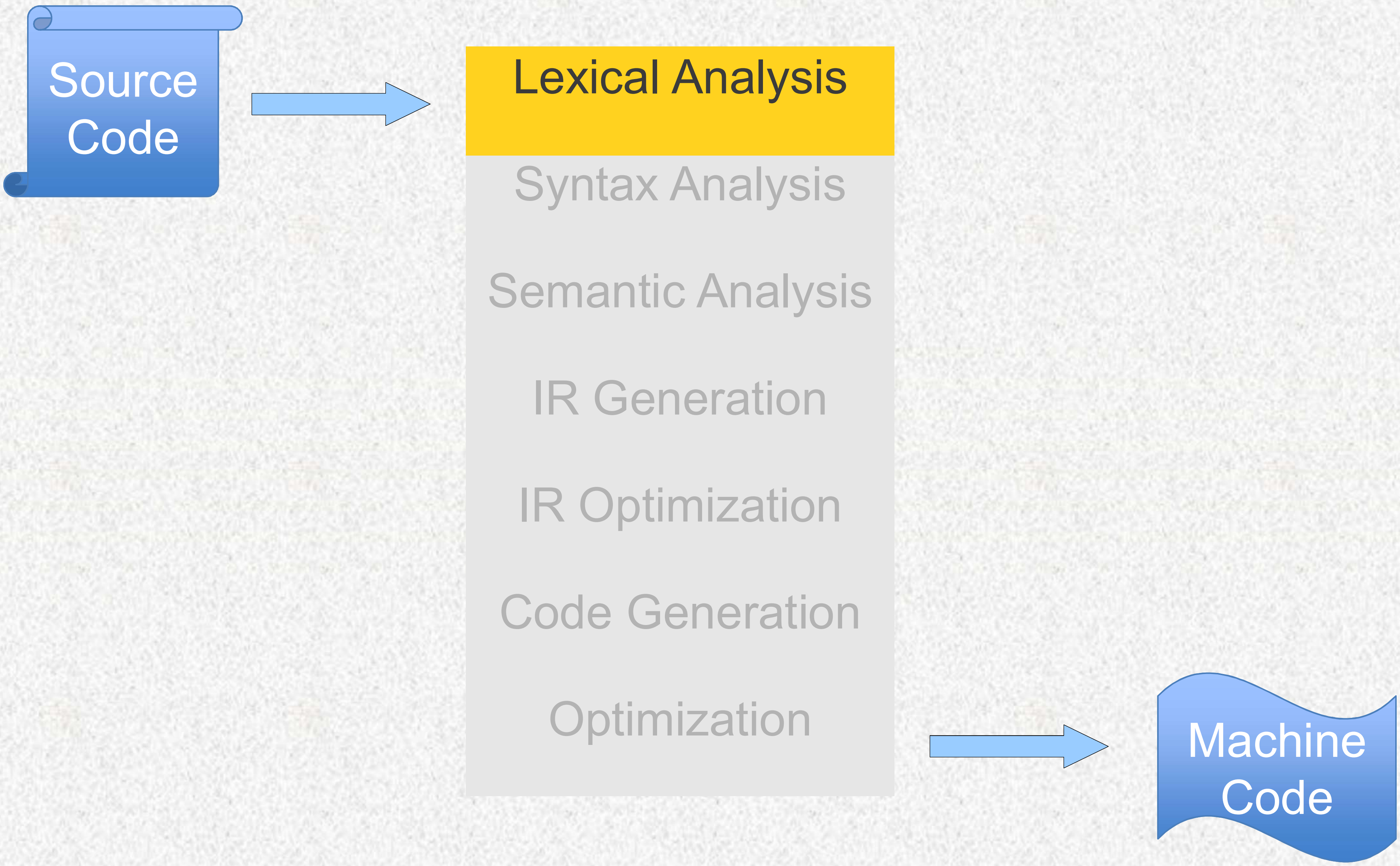
Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

Formalisms of tokens

Regular Expression

Finite Automaton

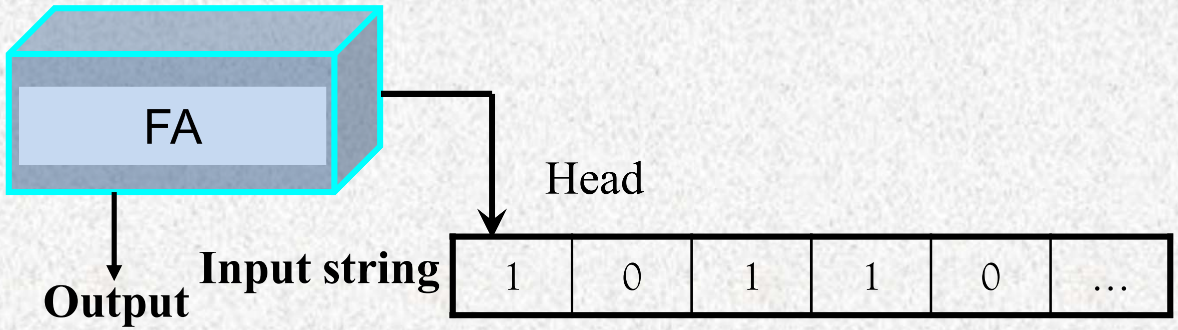
Implementing Regular Expressions

- Regular expressions can be implemented using **finite automata**.
  - Regular expressions = specification
  - Finite automata = implementation
- There are two main kinds of finite automata:
  - **NFA**s (**nondeterministic** finite automata)
  - **DFA**s (**deterministic** finite automata

Finite Automatons

- A finite automaton is a 5-tuple (S,Σ,δ,s_0,F)
  - A set of states S --- nodes
  - An input alphabet Σ
  - A transition function $\delta(S_i, a)=S_j$
  - A start state S_0
  - A set of accepting states $F \subseteq S$

Finite Automatons

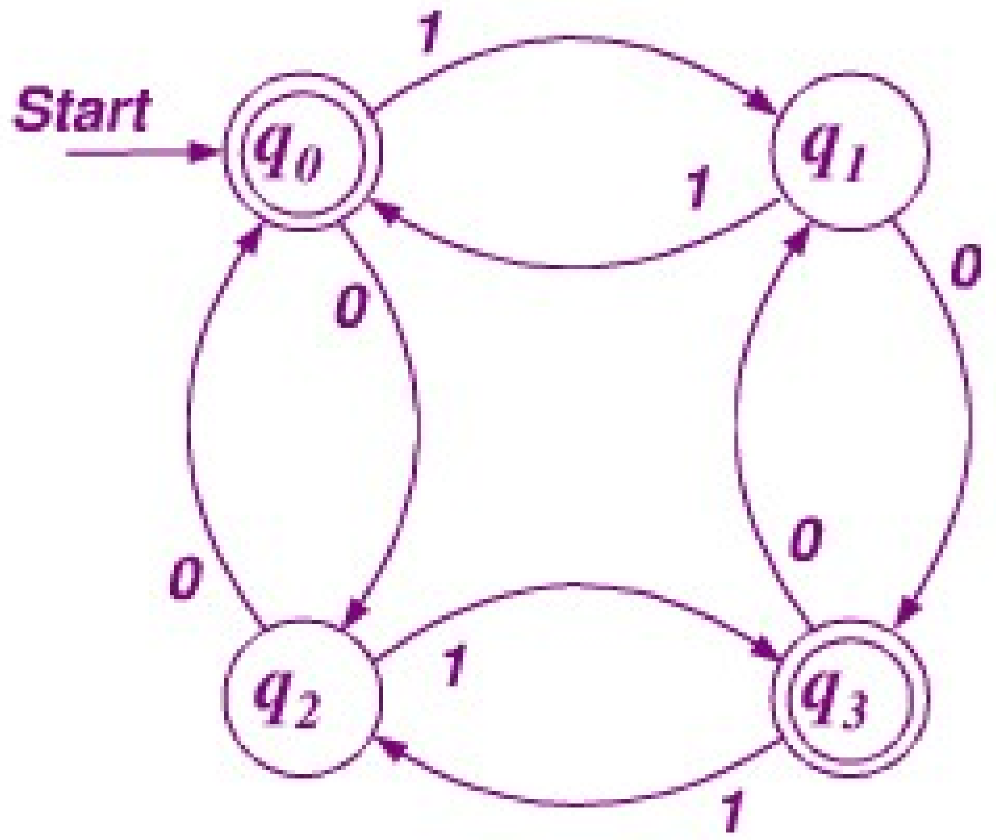


- Input: a string
- Output： accept if the scanning of input string reaches its EOF and the FA reaches an accepting state; reject otherwise

Strings accepted by an FA

- An FA *accepts* an input string *x* **iff** there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph

A More Complex Automaton

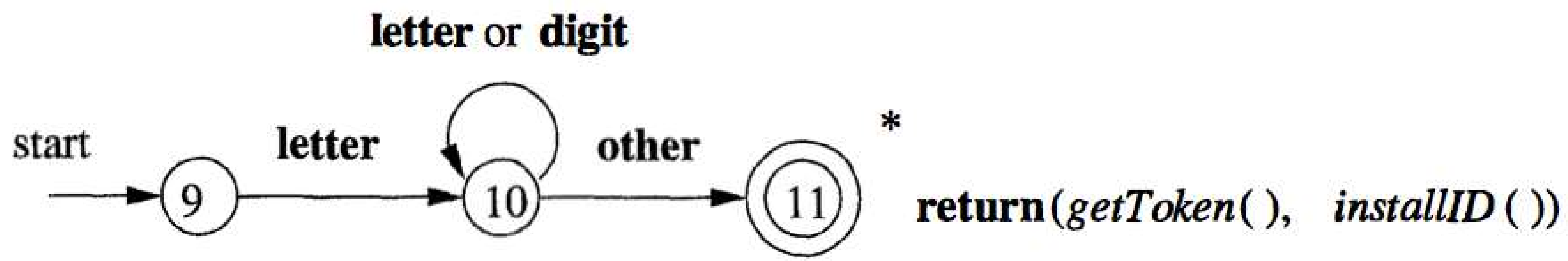


"1010": accept

"101": reject

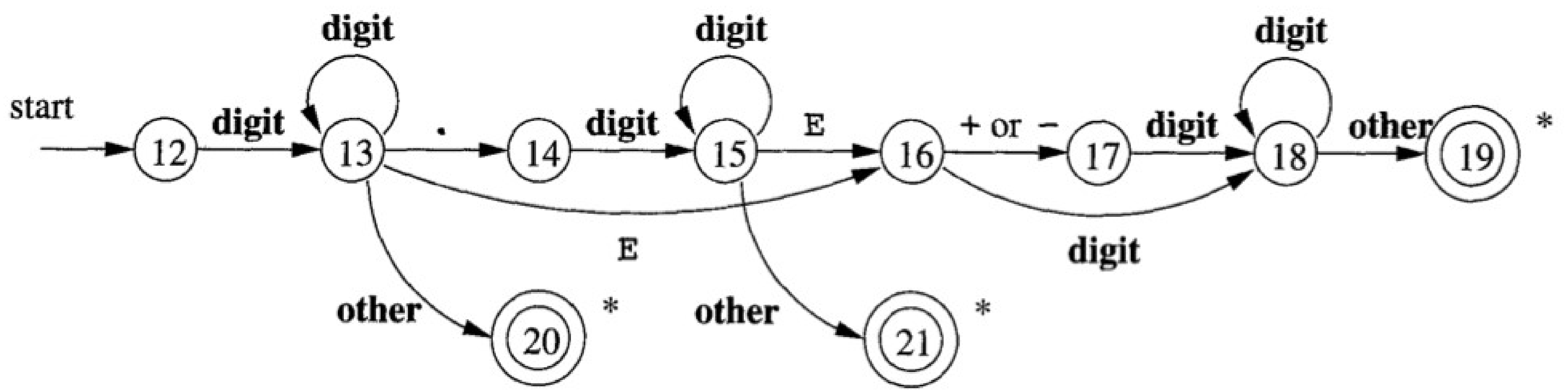
A state transition from one state to another on the path is called a *move*

A More Complex Automaton



h	i	1	2	3

A More Complex Automaton

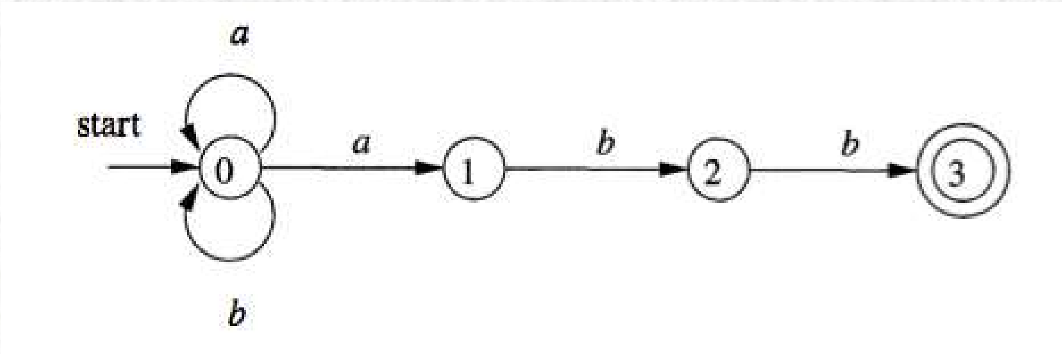


1	2	.	3	7	5

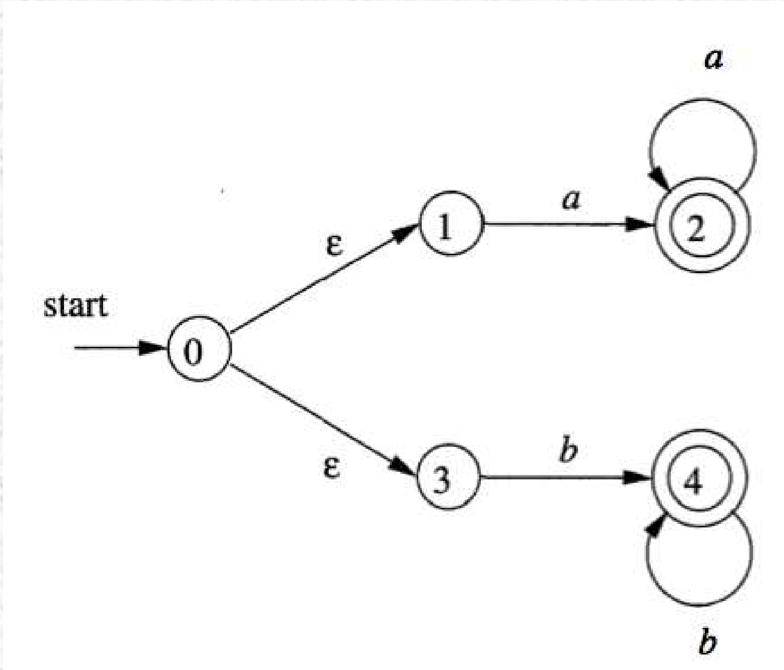
Language defined by an FA

- The *language defined by* an FA is the set of input strings it accepts, such as (**a**|**b**)***abb** for the example NFA

Languages defined by an FA



(a|b)*abb



aa*|bb*

Finite Automata

- Finite automata is a recognizer
- Given an input string, they simply say "yes" or "no" about each possible input string
  - **NFA**s (**nondeterministic** finite automata)
  - **DFA**s (**deterministic** finite automata
- To describe NFA or DFA, we have two methods
  - Transition diagram
  - Transition table

Nondeterministic Finite Automata (NFA)

- Definition: an NFA is a 5-tuple (S,Σ,δ,s_0,F) where
  - S is a finite set of *states*
  - Σ is a finite set of *input symbol alphabet*
  - δ is a *mapping* from $S \times \Sigma \cup \{\varepsilon\}$ to a set of states
  - $S_0 \subseteq S$ is the set of *start states*
  - $F \subseteq S$ is the set of *accepting* (or *final*) *states*

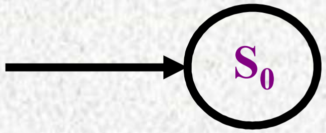
Nondeterministic Finite Automata (NFA)

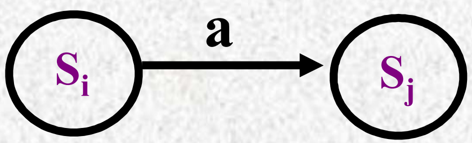
- Transition Graph

Node: **State**

- Non-terminal state: 

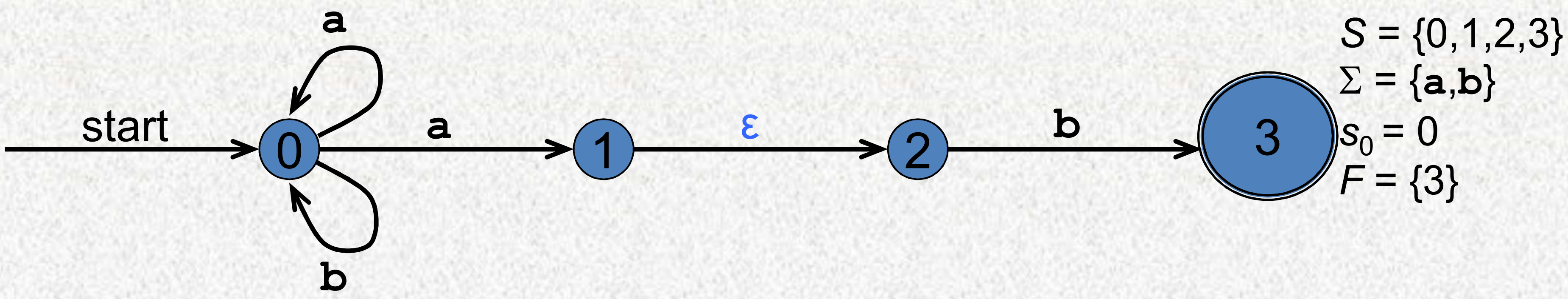
- Terminal state: 

- Starting state: 

Edge: **state transition** $f(S_i,a)=S_j$ 

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



Nondeterministic Finite Automata (NFA)

- **Transit table**
  - Line：State
    - Starting state：in general, the first line，or label “+”;
    - Terminal state: “*” or “-” or “⊥”;
  - Column：All symbols in ∑
  - Cell：state transition mapping

Transition Table

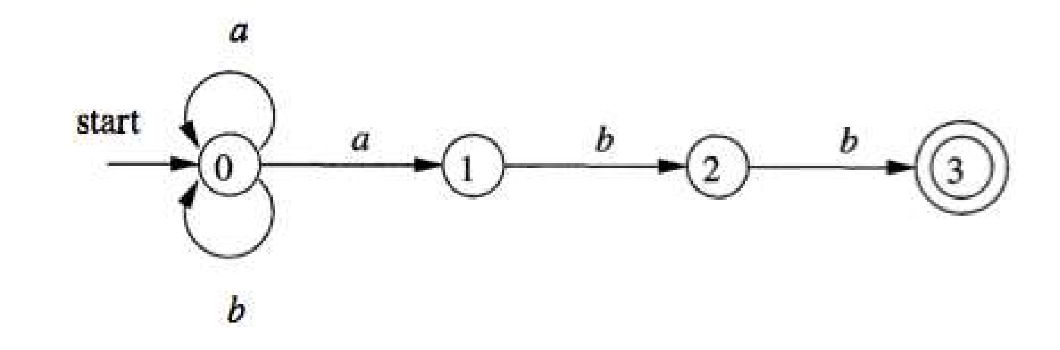
- The mapping δ of an NFA can be represented in a *transition table*

$\delta(0,\mathtt{a}) = \{0,1\}$
$\delta(0,\mathtt{b}) = \{0\}$
$\delta(1,\mathtt{b}) = \{2\}$
$\delta(2,\mathtt{b}) = \{3\}$

→

State	*Input* **a**	*Input* **b**
0	{0,1}	{0}
1		{2}
2		{3}

NFA Example 2



Transition Table

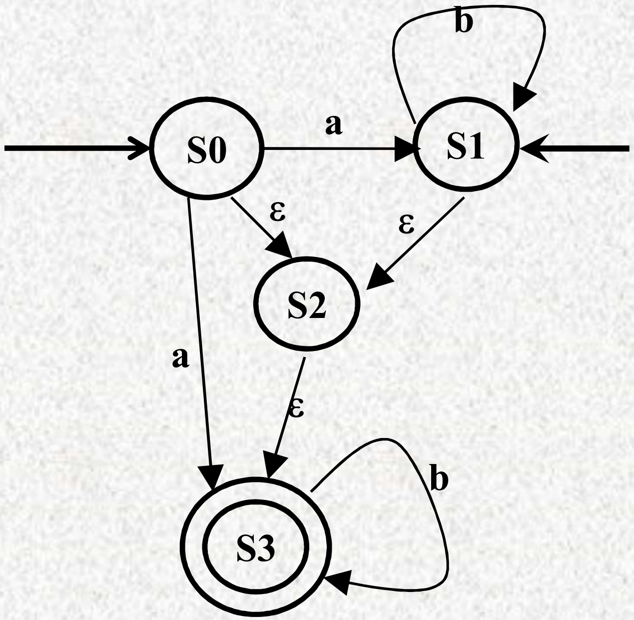
STATE	a	b	ϵ
0	$\{0,1\}$	$\{0\}$	$\emptyset$
1	$\emptyset$	$\{2\}$	$\emptyset$
2	$\emptyset$	$\{3\}$	$\emptyset$
3	$\emptyset$	$\emptyset$	$\emptyset$

Acceptance of input strings

$$0 \xrightarrow{a} 0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3$$

$$0 \xrightarrow{a} 0 \xrightarrow{a} 0 \xrightarrow{b} 0 \xrightarrow{b} 0$$

NFA Example 3



	a	b	ε
S0+	{S1,S3}		{S2}
S1+		{S1}	{S2}
S2			{S3}
S3-		{S3}	

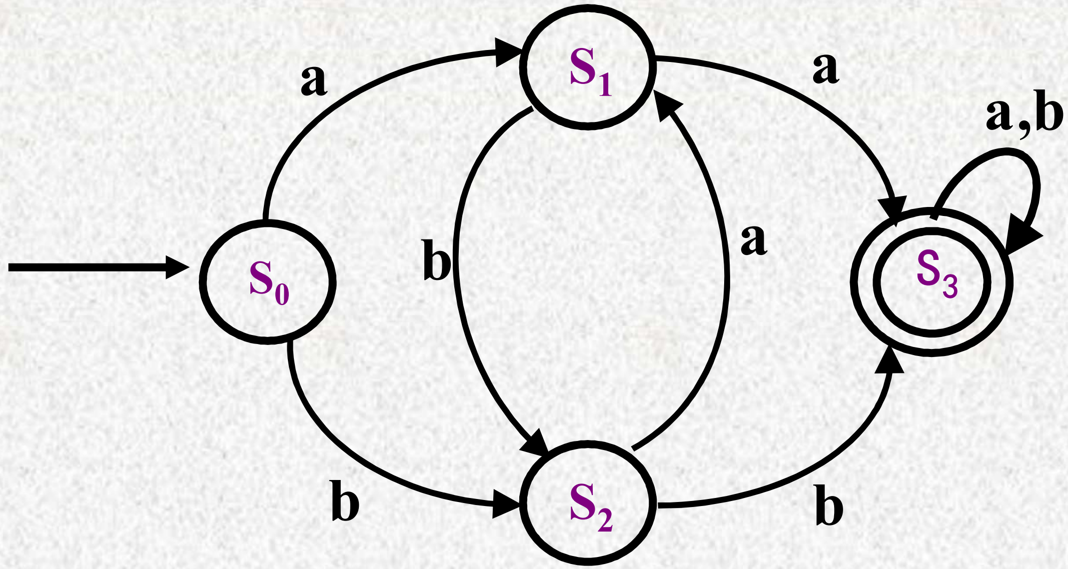
Deterministic Finite Automata (DFA)

- Definition: an DFA is a 5-tuple (S,Σ,δ,s_0,F), is a special case of NFA
  - There are no moves on input ε, and
  - For each state s and input symbol a, there is exactly one edge out of s labeled a.

Deterministic Finite Automata (DFA)

- DFA M=({S0, S1, S2, S3}, {a,b}, f, S0, {S3}), ：

f (S0, a)=S1	f (S2, a)=S1
f (S0, b)=S2	f (S2, b)= S3
f (S1, a)= S3	f (S3, a)= S3
f (S1, b)= S2	f (S3, b)= S3



Deterministic Finite Automata (DFA)

- For example, DFA $M=(\{0,1,2,3,4\},\{a,b\}, \delta,\{0\},\{3\})$
- $\delta(0, a) = 1$ $\delta(0, b) = 4$

$\delta(1, a) = 4$ $\delta(1, b) = 2$

$\delta(2, a) = 3$ $\delta(2, b) = 4$

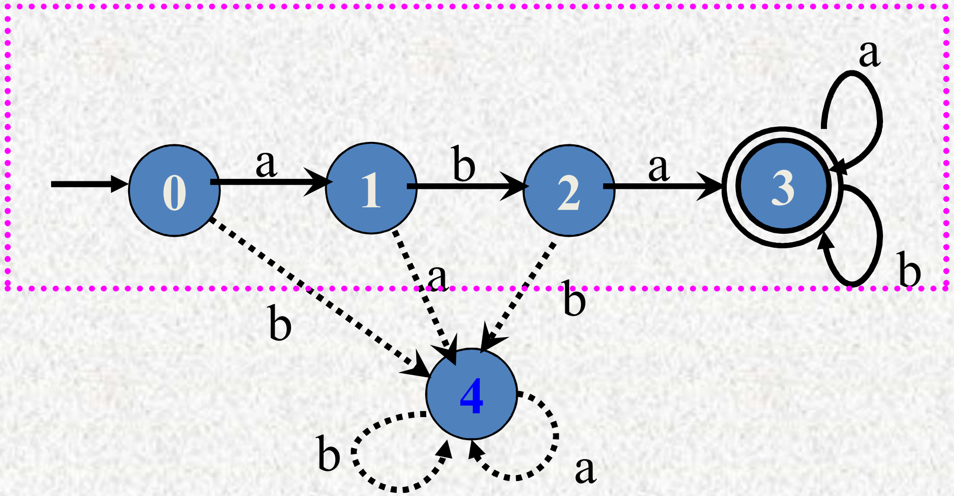
$\delta(3, a) = 3$ $\delta(3, b) = 3$

$\delta(4, a) = 4$ $\delta(4, b) = 4$

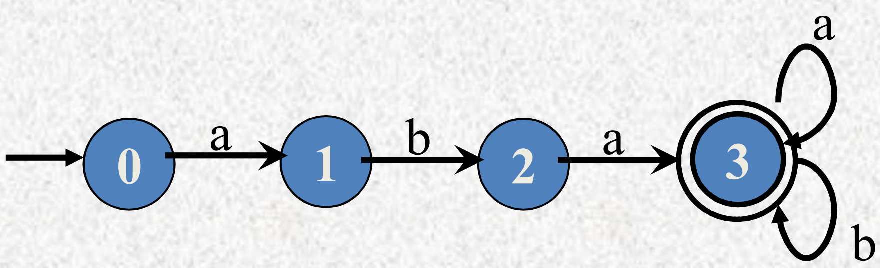
	a	b
0^+	1	4
1	4	2
2	3	4
3^-	3	3
4	4	4

Deterministic Finite Automata (DFA)

	a	b
0+	1	4
1	4	2
2	3	4
3−	3	3
4	4	4



Deterministic Finite Automata (DFA)



	a	b
0+	1	⊥
1	⊥	2
2	3	⊥
3⁻	3	3

↔

	a	b
0+	1	
1		2
2	3	
3⁻	3	3

Deterministic Finite Automata (DFA)

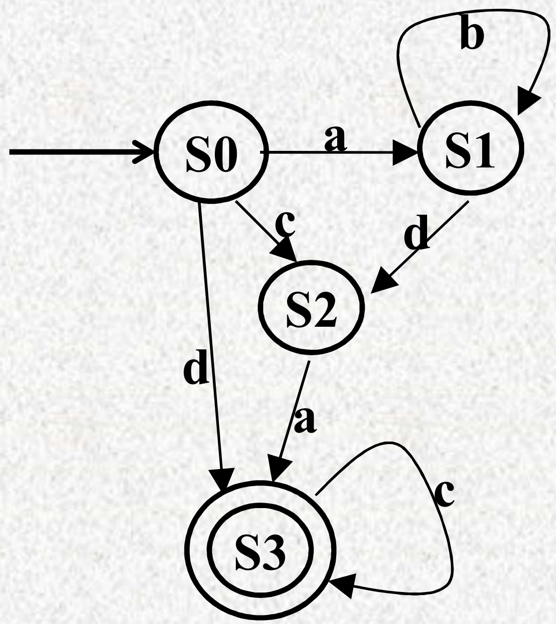
∑: {a, b, c, d}

S: {S0, S1, S2, S3}

Start: S0

Terminal: {S3}

f: {(S0,a)→ S1, (S0,c)→S2,
(S0,d)→S3, (S1,b)→S1,
(S1,d)→S2, (S2,a)→S3,
(S3, c)→S3}



NFA v.s. DFA

NFA v.s. DFA

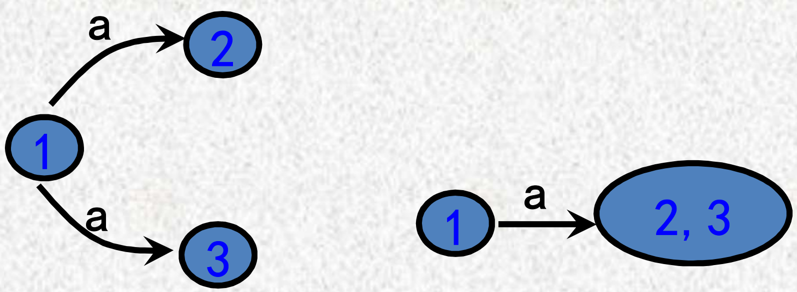
	DFA	NFA
Initial	Single starting state	A set of starting states
ε dege	Not allowed	Allowed
δ(S, a)	S' or ⊥	{S1, …, Sn} or ⊥
Implementation	Deterministic	Nondeterministic

- DFA accepts an input string with only one path
- NFA accepts an input string with possibly multiple paths

Construct DFA from NFA

- Construct DFA from NFA
  - For any NFA, there exists an equivalent DFA
  - Idea of construction: eliminate the uncertainty
  - Merge N states in NFA into one single state
    - Eliminate ε

    - Eliminate multiple mapping

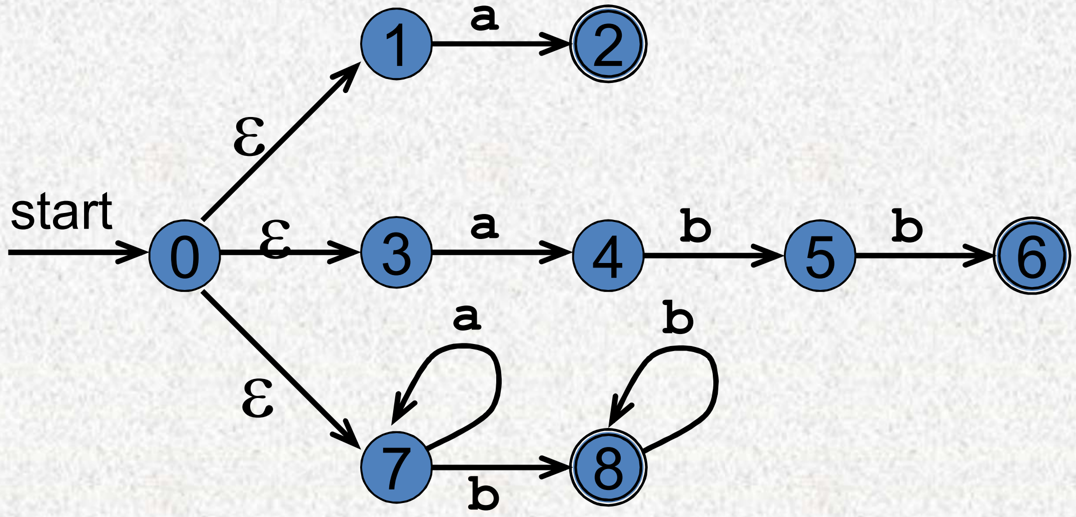


Construct DFA from NFA

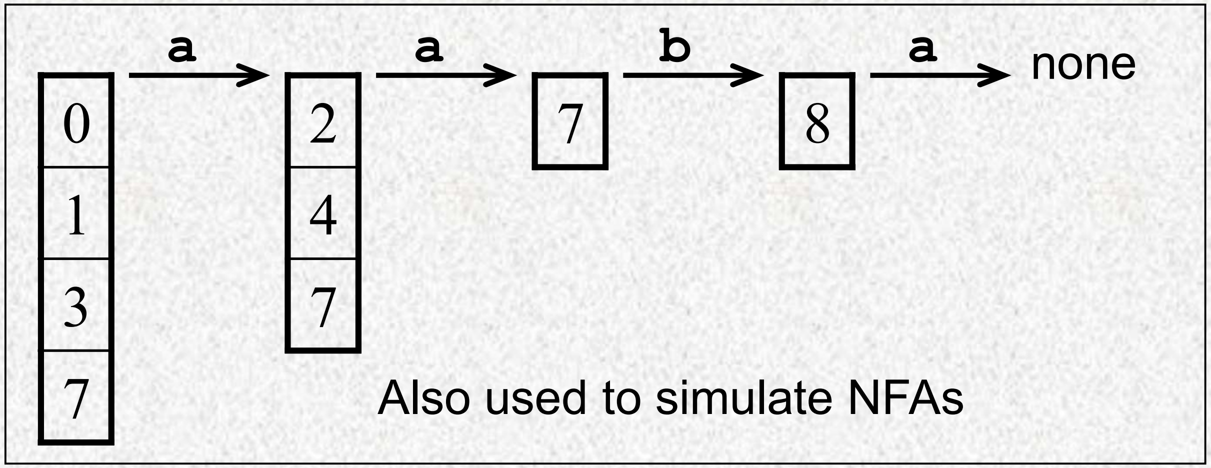
- **INPUT**: An NFA N.
- **OUTPUT**: A DFA D accepting the same language as N.
- **METHOD**: The algorithm constructs a transition table Dtran for D. Each state of D is a set of NFA states, and we construct Dtran so D will simulate "in parallel" all possible moves N can make on a given input string.

OPERATION	DESCRIPTION
ϵ-*closure*(s)	Set of NFA states reachable from NFA state s on ϵ-transitions alone.
ϵ-*closure*(T)	Set of NFA states reachable from some NFA state s in set T on ϵ-transitions alone; $= \cup_{s \text{ in } T}\ \epsilon\text{-}closure(s)$.
move(T, a)	Set of NFA states to which there is a transition on input symbol a from some state s in T.

ε-*closure* and *move* Examples



ε-*closure*({0}) = {0,1,3,7}
move({0,1,3,7},**a**) = {2,4,7}
ε-*closure*({2,4,7}) = {2,4,7}
move({2,4,7},**a**) = {7}
ε-*closure*({7}) = {7}
move({7},**b**) = {8}
ε-*closure*({8}) = {8}
move({8},**a**) = $\emptyset$



Simulating the NFA

Algorithm 3.22: Simulating an NFA.

INPUT: An input string x terminated by an end-of-file character **eof**. An NFA N with start state s_0, accepting states F, and transition function *move*.

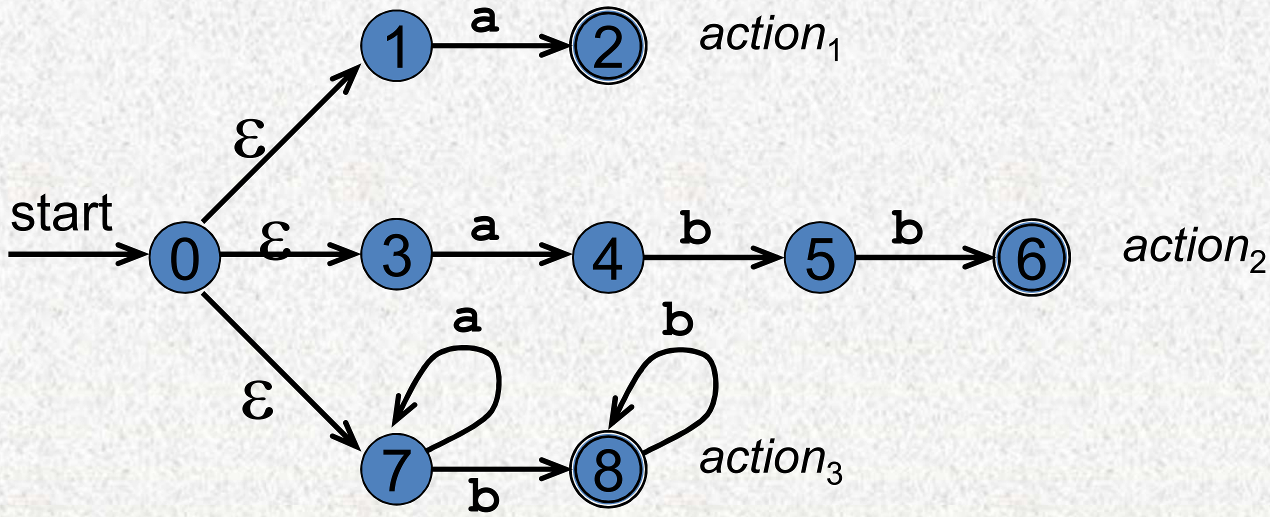
OUTPUT: Answer "yes" if M accepts x; "no" otherwise.

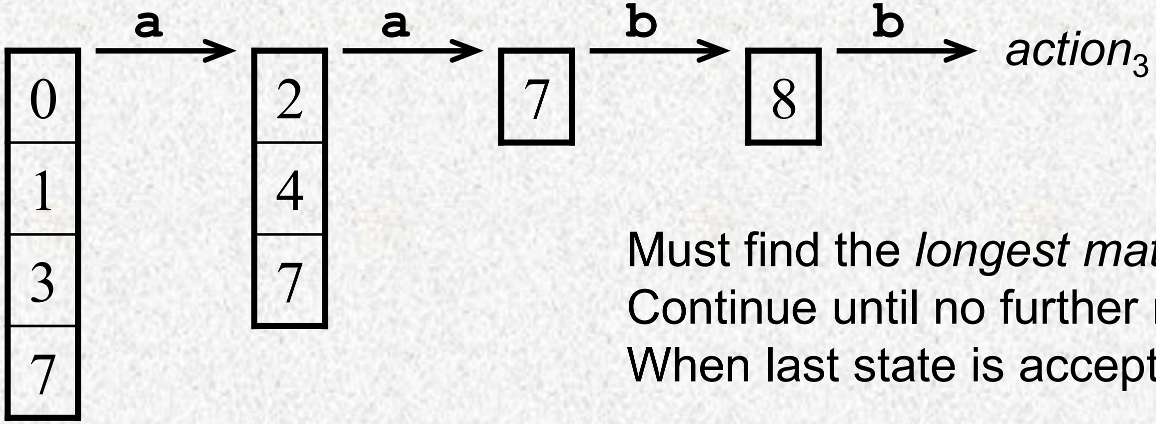
METHOD: The algorithm keeps a set of current states S, those that are reached from s_0 following a path labeled by the inputs read so far. If c is the next input character, read by the function *nextChar*(), then we first compute $move(S, c)$ and then close that set using ϵ-*closure*(). The algorithm is sketched in Fig. 3.37. □

```
S = ε-closure(s0);
c = nextChar();
while ( c != eof ) {
        S = ε-closure(move(S,c));
        c = nextChar();
}
if ( S ∩ F != ∅ ) return "yes";
else return "no";
```

Figure 3.37: Simulating an NFA

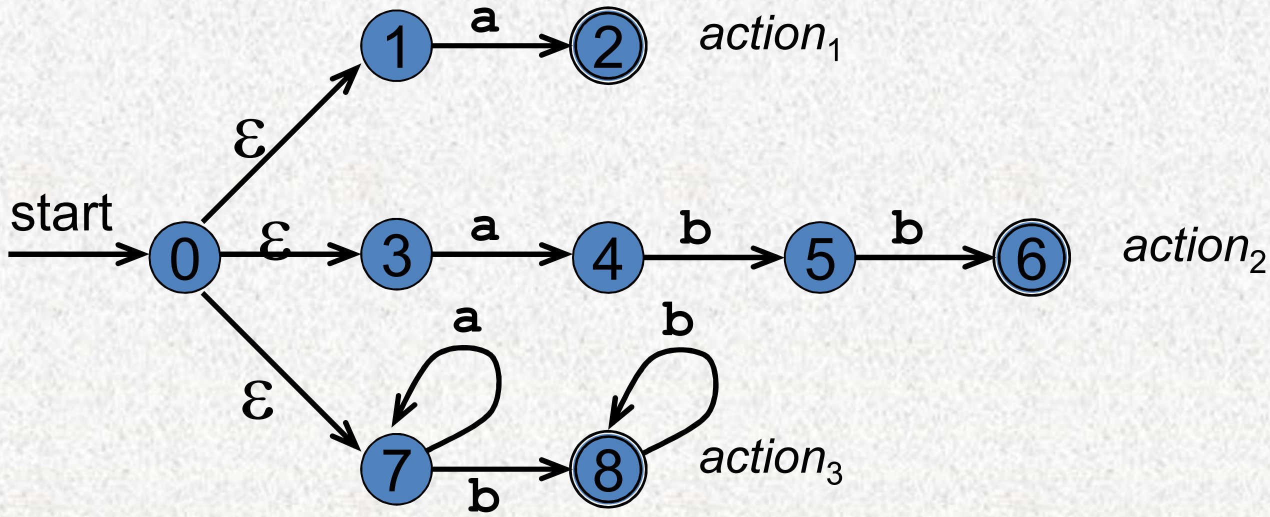
Simulating a NFA Example 1

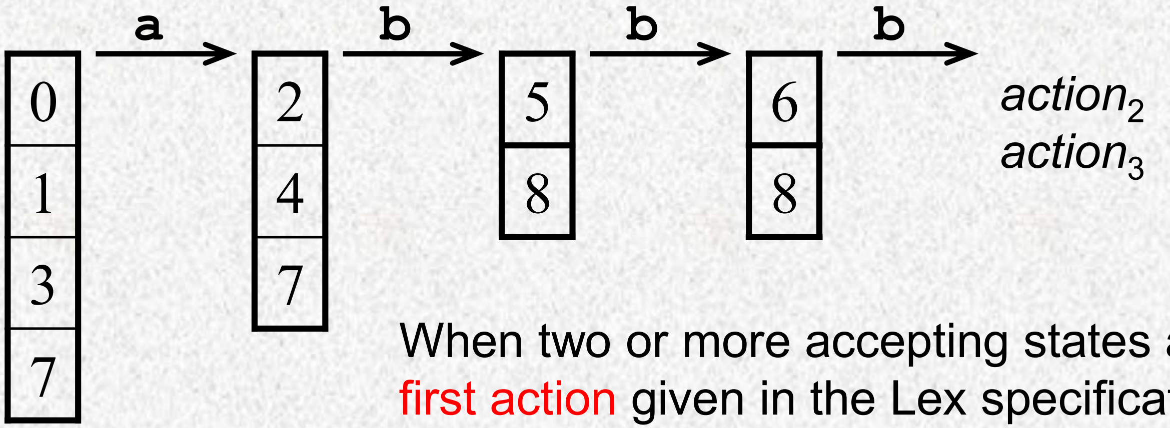




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating a NFA Example 2





When two or more accepting states are reached, the first action given in the Lex specification is executed

The Subset Construction Algorithm

- NFAs can be in many states at once, while DFAs can only be in a single state at a time.
- Key idea: **Make the DFA simulate the NFA**.
- Have the states of the DFA correspond to the *sets of states* of the NFA.
- Transitions between states of DFA correspond to transitions between *sets of states* in the NFA.

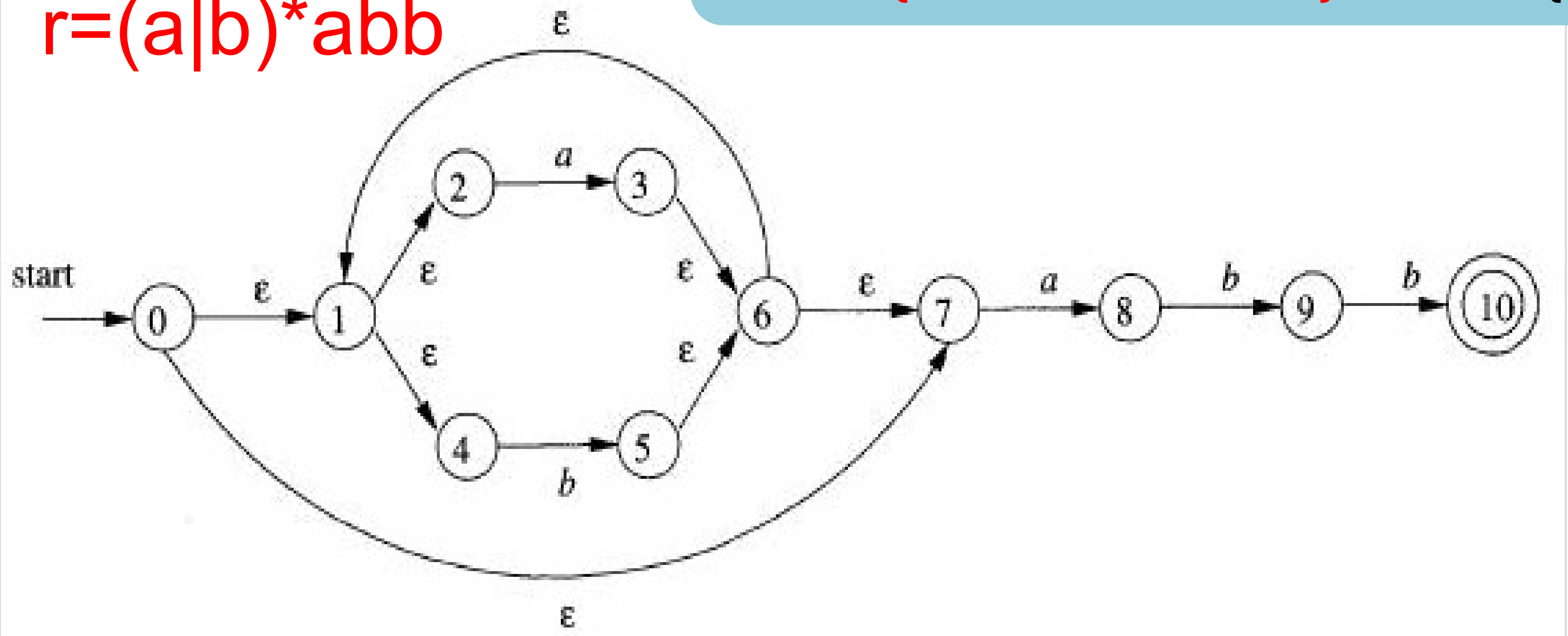
The Subset Construction Algorithm

```
initially, ε-closure(s0) is the only state in Dstates, and it is unmarked;
while ( there is an unmarked state T in Dstates ) {
        mark T;
        for ( each input symbol a ) {
                U = ε-closure(move(T, a));
                if ( U is not in Dstates )
                        add U as an unmarked state to Dstates;
                Dtran[T, a] = U;
        }
}
```

Subset Construction Example 1

First，Initial state of NFA is ε-closure(0)，
i.e. A={0，1，2，4，7}， Σ = {a,b}

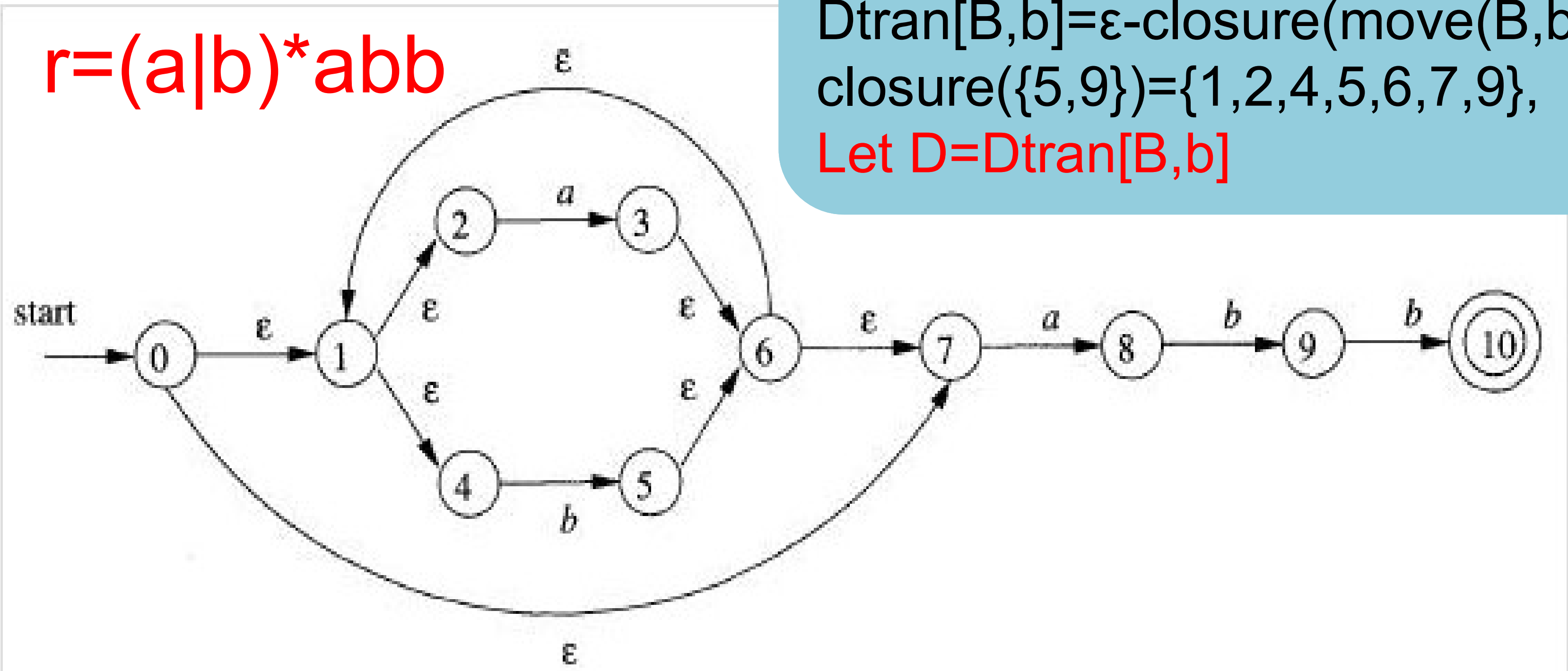
r=(a|b)*abb



Dtran[A,a]=ε-closure(move(A,a))=ε-closure({3,8})={1,2,3,4,6,7,8},
Let B=Dtran[A,a]
Dtran[A,b]=ε-closure(move(A,b))=ε-closure({5})={1,2,4,6,7},
Let C=Dtran[A,b]

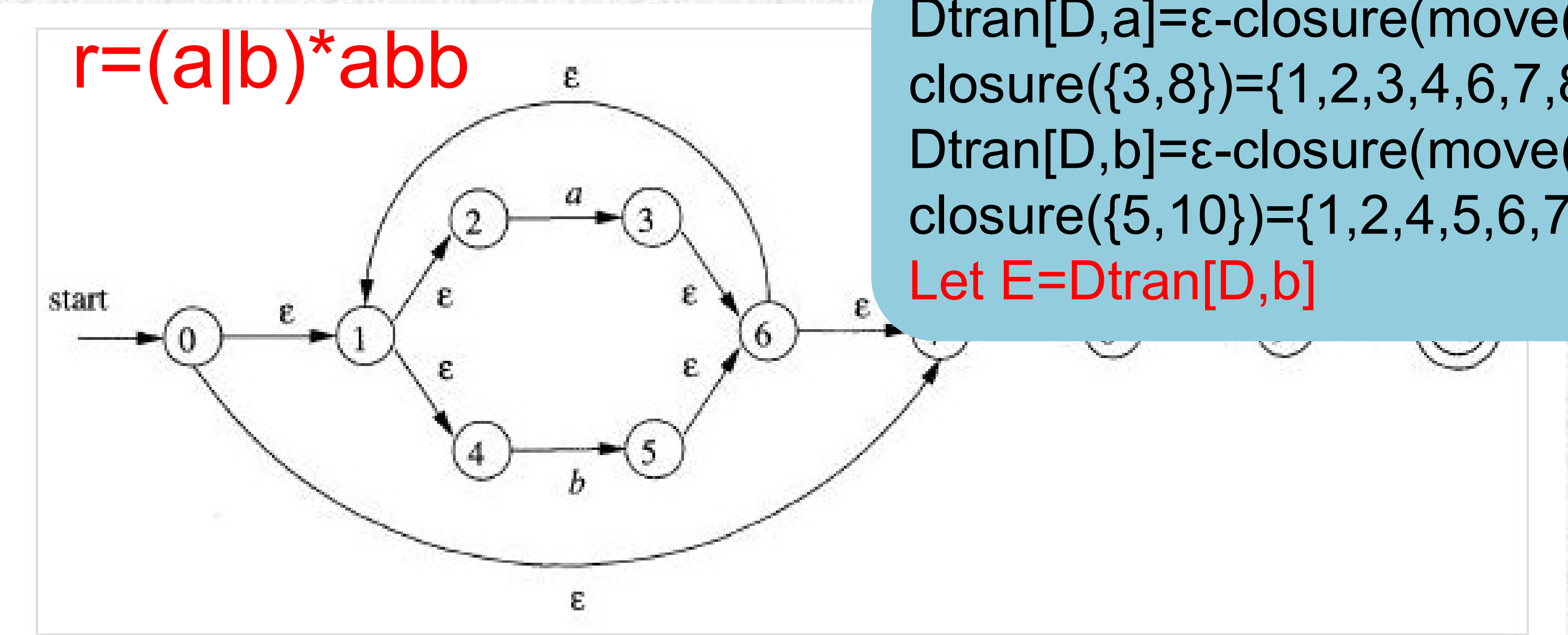
Subset Construction Example 1

Dtran[B,a]=ε-closure(move(B,a))=ε-closure({3,8})={1,2,3,4,6,7,8}=B
Dtran[B,b]=ε-closure(move(B,b))=ε-closure({5,9})={1,2,4,5,6,7,9},
Let D=Dtran[B,b]



Dtran[C,a]=ε-closure(move(C,a))=ε-closure({3,8})={1,2,3,4,6,7,8}=B
Dtran[C,b]=ε-closure(move(C,b))=ε-closure({5})={1,2,4,6,7}=C

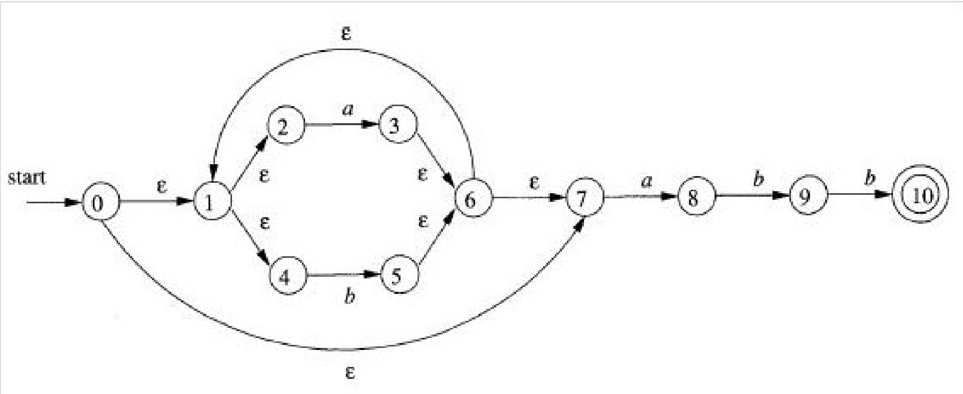
Subset Construction Example 1



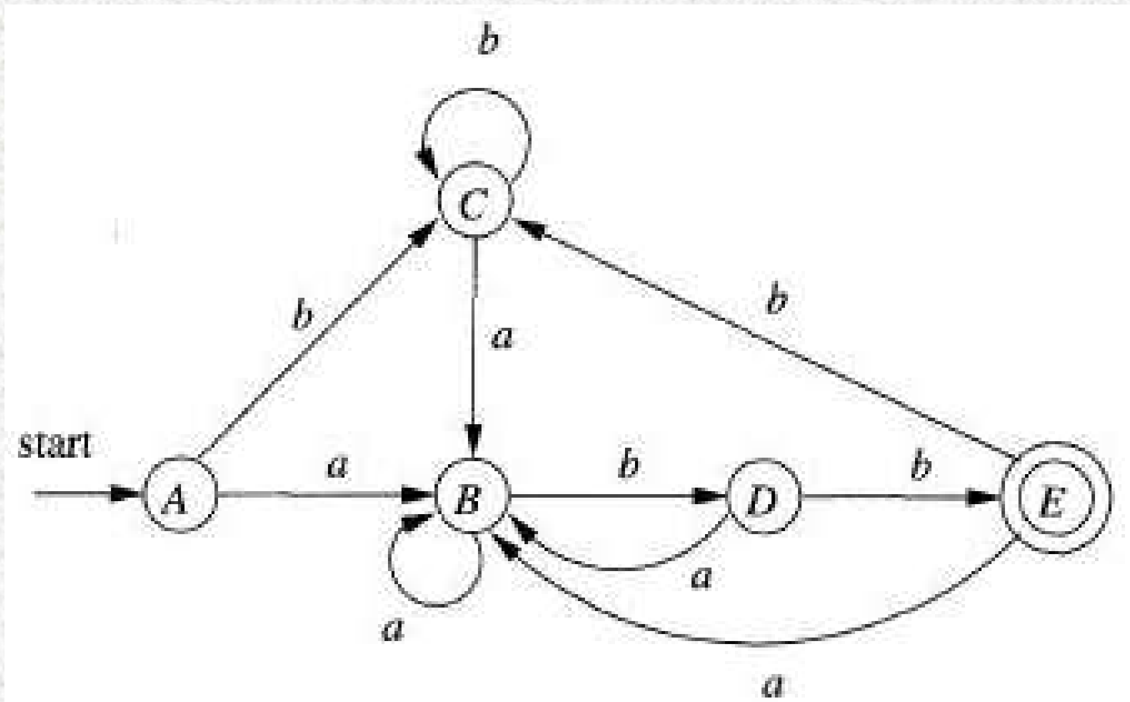
Dtran[D,a]=ε-closure(move(D,a))=ε-closure({3,8})={1,2,3,4,6,7,8}=B
Dtran[D,b]=ε-closure(move(D,b))=ε-closure({5,10})={1,2,4,5,6,7,10},
Let E=Dtran[D,b]

Dtran[E,a]=ε-closure(move(E,a))=ε-closure({3,8})={1,2,3,4,6,7,8}=B
Dtran[E,b]=ε-closure(move(E,b))=ε-closure({5})={1,2,4,6,7}=C

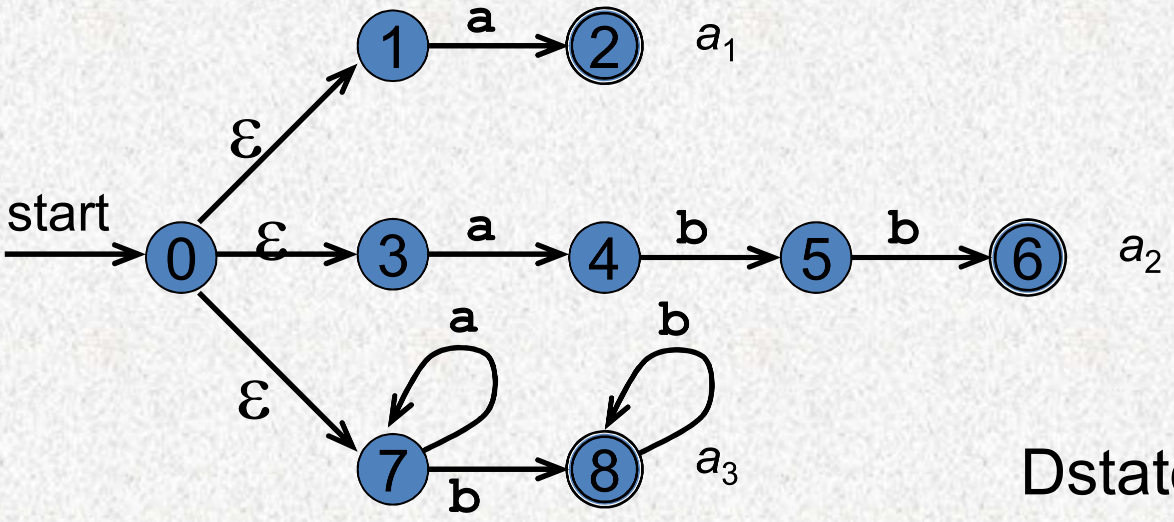
Subset Construction Example 1

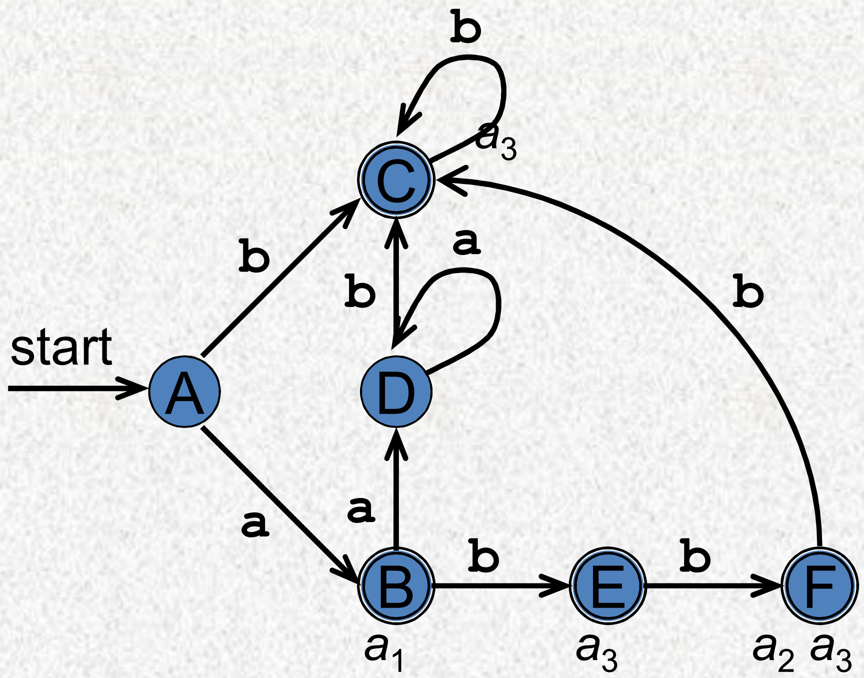


NFA STATE	DFA STATE	a	b
$\{0,1,2,4,7\}$	A	B	C
$\{1,2,3,4,6,7,8\}$	B	B	D
$\{1,2,4,5,6,7\}$	C	B	C
$\{1,2,4,5,6,7,9\}$	D	B	E
$\{1,2,3,5,6,7,10\}$	E	B	C



Subset Construction Example 2





Dstates

A = {0,1,3,7}

B = {2,4,7}

C = {8}

D = {7}

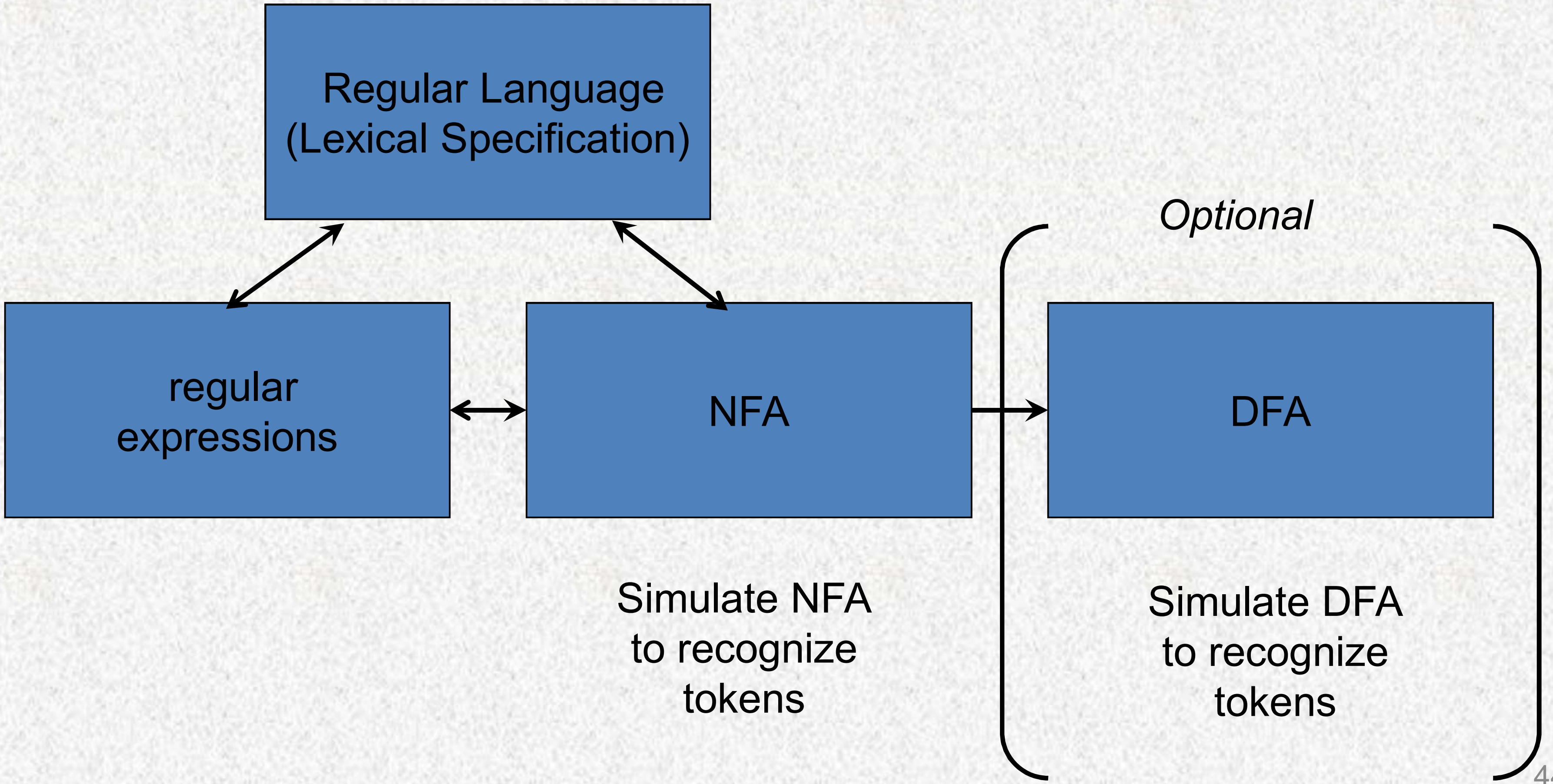
E = {5,8}

F = {6,8}

RE to NFA/DFA

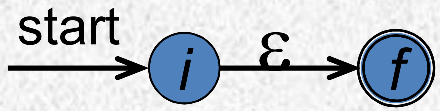
Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA

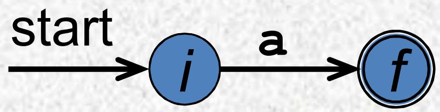


From Regular Expression to NFA (Thompson's Construction)

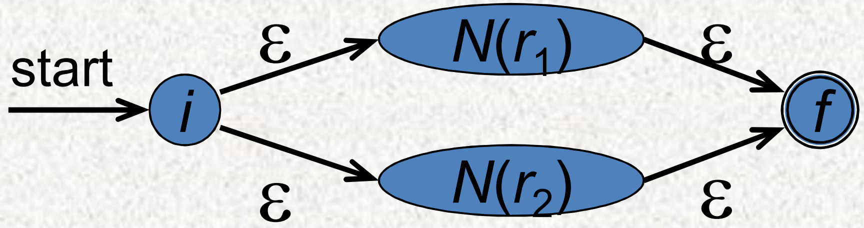
ε



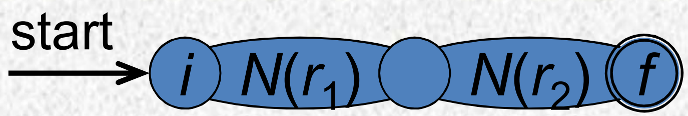
a $\in \Sigma$



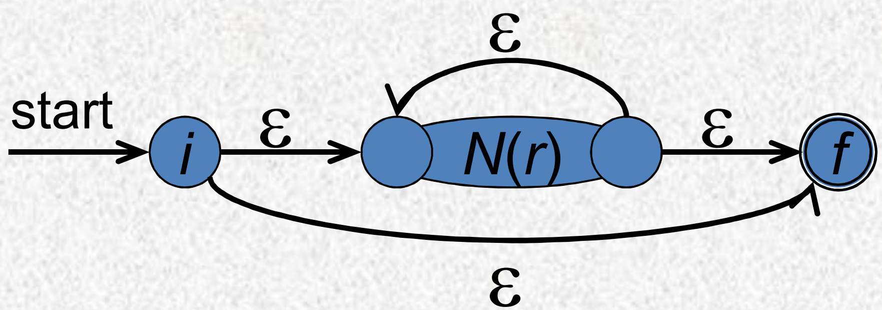
$r_1 \mid r_2$



$r_1 r_2$



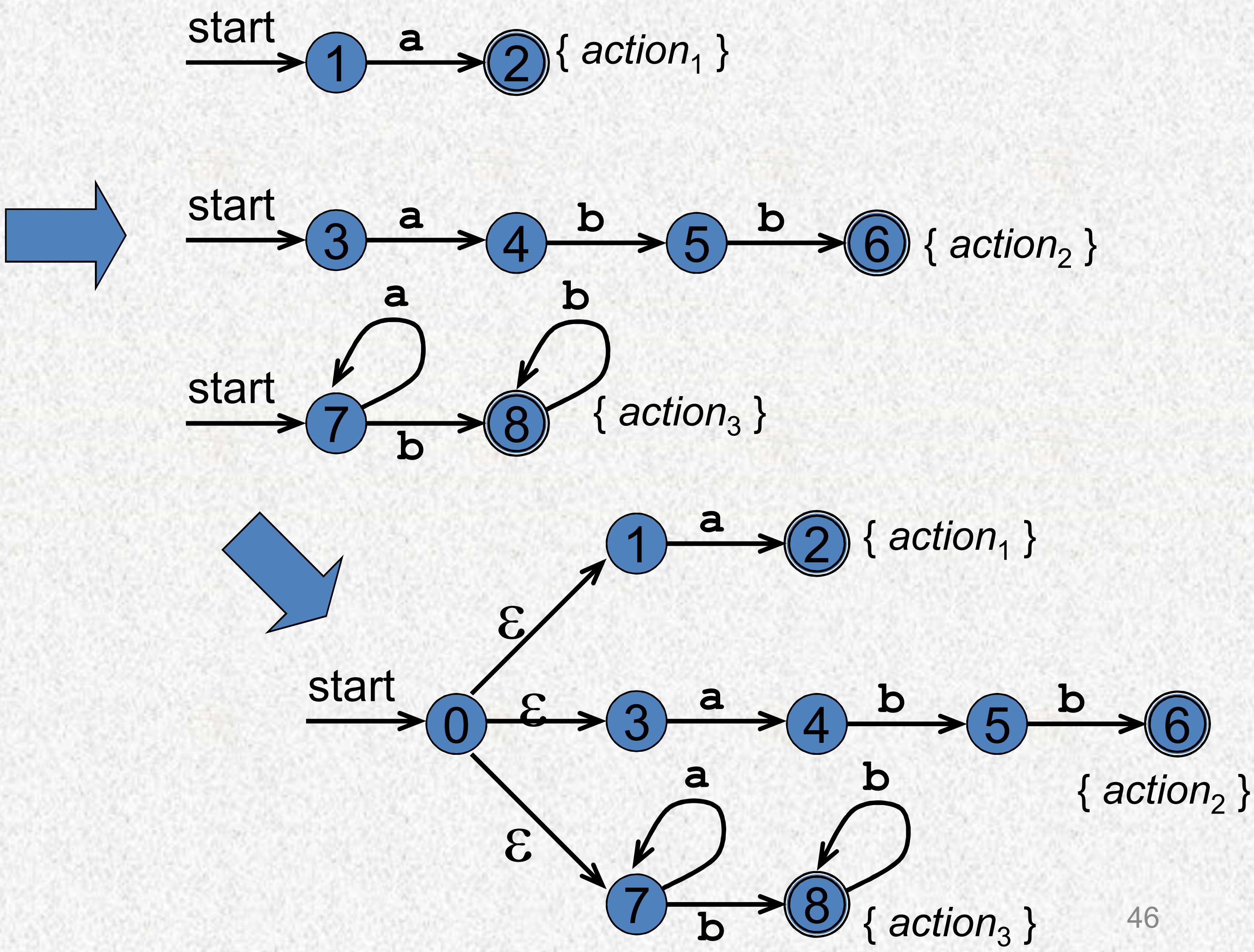
r^*



Combining the NFAs of a Set of Regular Expressions

a | abb | a*b+

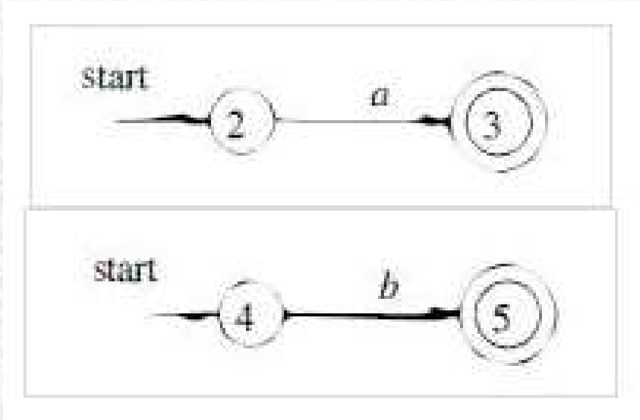
a	{ *action*$_1$ }
abb	{ *action*$_2$ }
a*b+	{ *action*$_3$ }



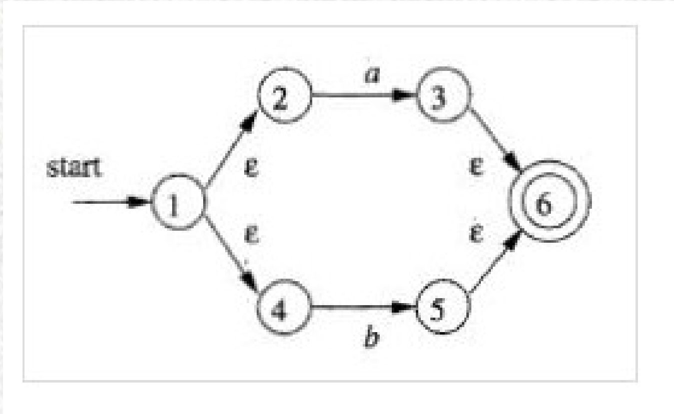
Combining the NFAs of a Set of Regular Expressions

r=(a|b)*abb

r1 = a，r2=b, we have NFA:



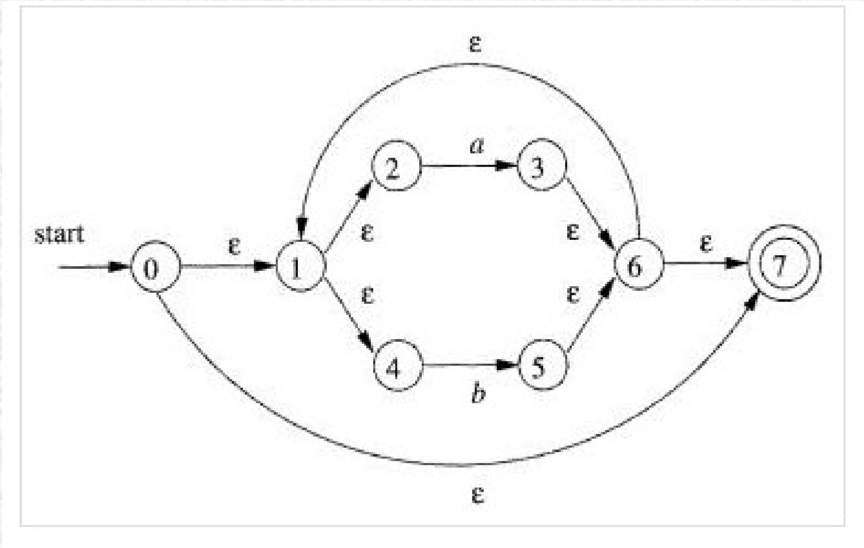
r3 = r1|r2, we have NFA:



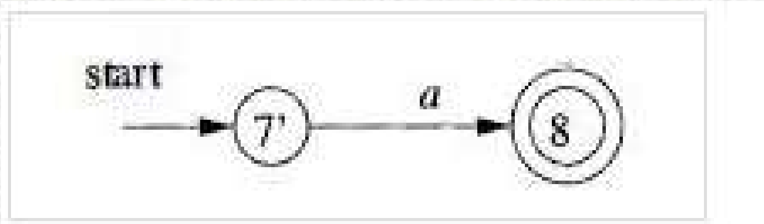
Combining the NFAs of a Set of Regular Expressions

r=(a|b)*abb

r5 = r3*, we have NFA:



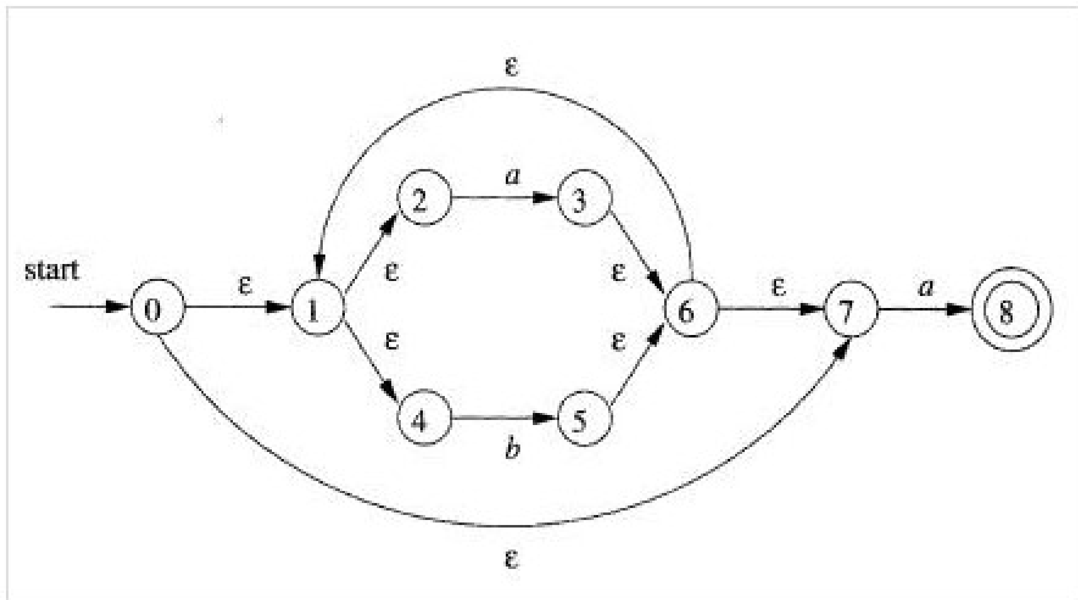
r6 = a, we have NFA:



Combining the NFAs of a Set of Regular Expressions

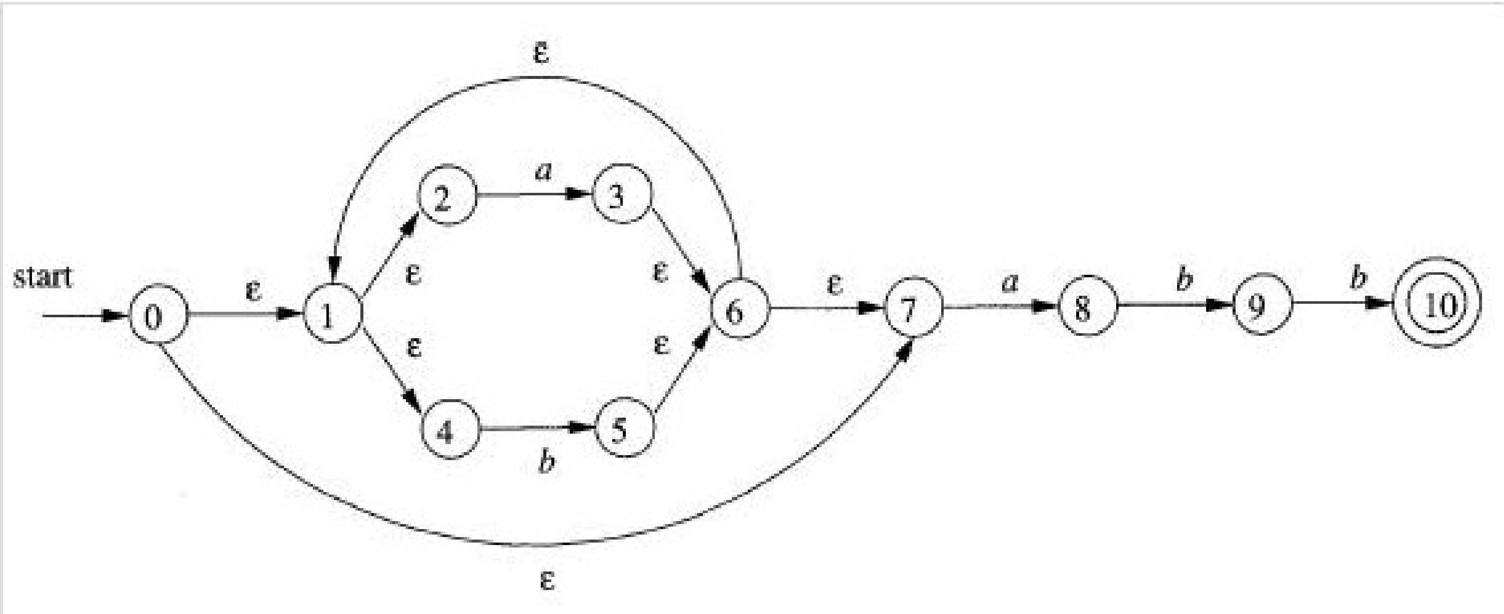
r=(a|b)*abb

r7 = r5r6, we have NFA:



Combining the NFAs of a Set of Regular Expressions

r=(a|b)*abb

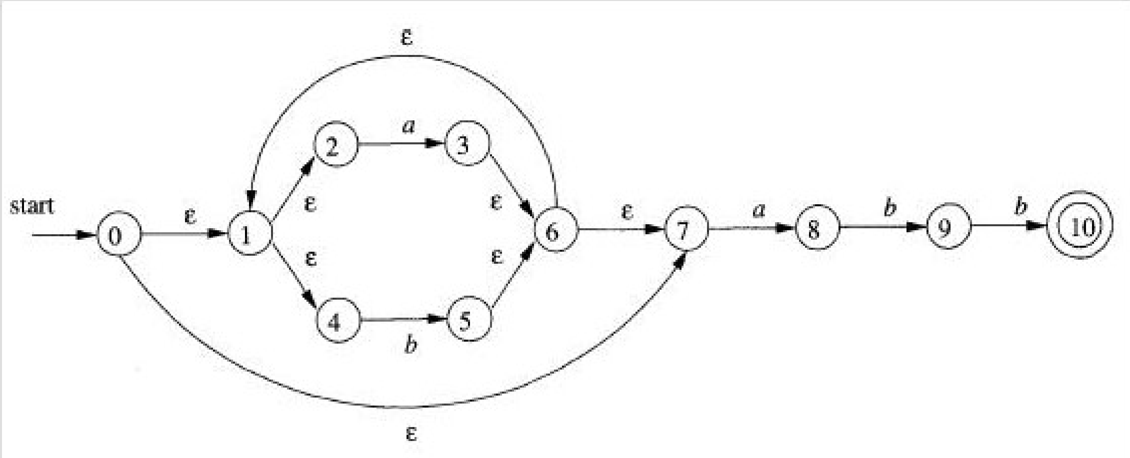


From NFA to DFA

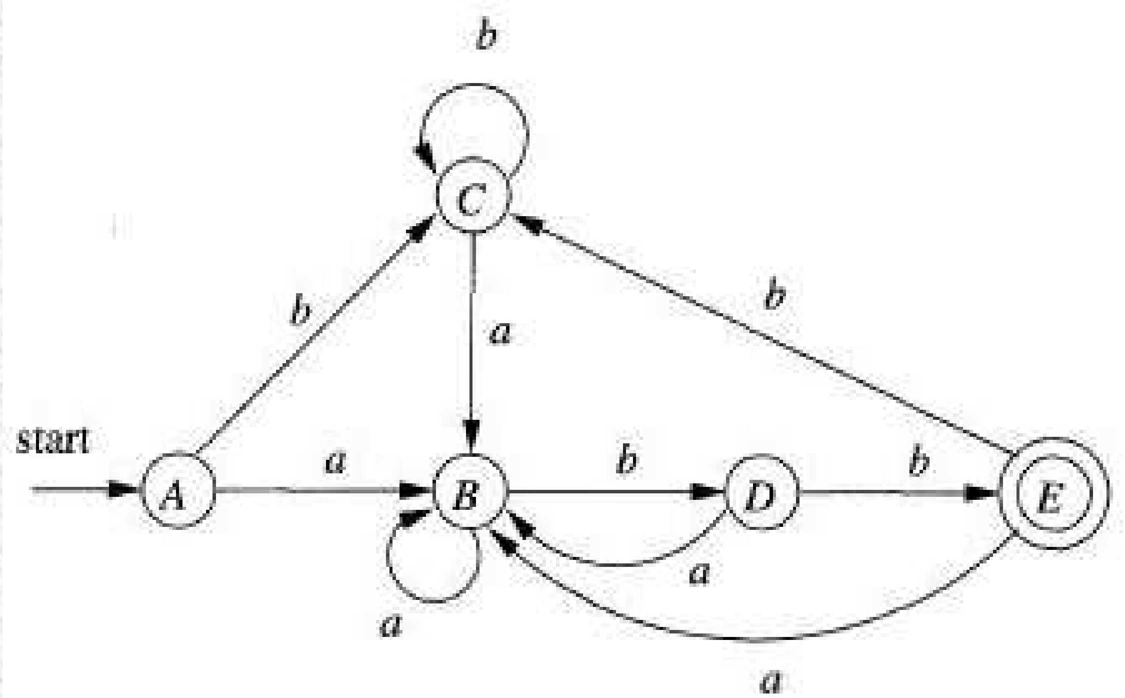
The Subset Construction Algorithm

```
initially, ε-closure(s0) is the only state in Dstates, and it is unmarked;
while ( there is an unmarked state T in Dstates ) {
        mark T;
        for ( each input symbol a ) {
                U = ε-closure(move(T, a));
                if ( U is not in Dstates )
                        add U as an unmarked state to Dstates;
                Dtran[T, a] = U;
        }
}
```

From NFA to DFA

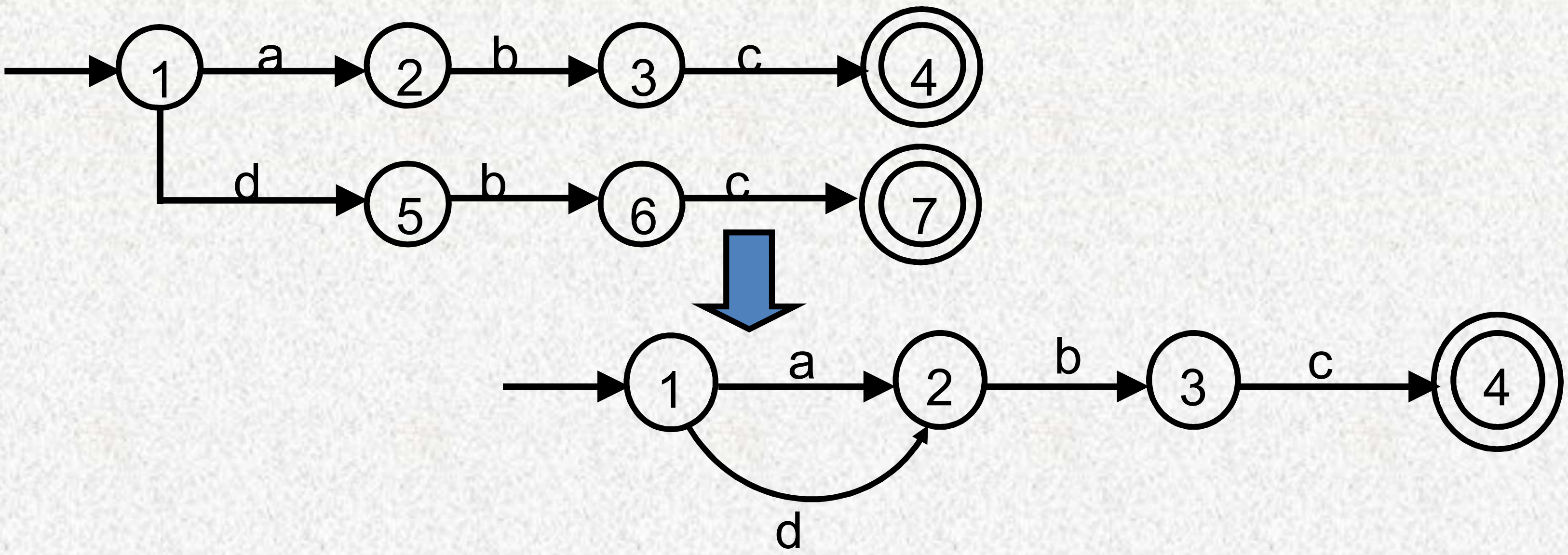


NFA STATE	DFA STATE	a	b
{0, 1, 2, 4, 7}	A	B	C
{1, 2, 3, 4, 6, 7, 8}	B	B	D
{1, 2, 4, 5, 6, 7}	C	B	C
{1, 2, 4, 5, 6, 7, 9}	D	B	E
{1, 2, 3, 5, 6, 7, 10}	E	B	C



Minimizing DFA

After conversion from NFA, the DFA may contain some equivalent states, which lead to low efficiency in the analysis



Minimizing DFA

- ***Lots*** of methods
- All involve finding **equivalent states**:
  - States that go to equivalent states under all inputs (sounds recursive)
- We will use the ***Partitioning Method***

Minimizing DFA

- Step 1
  - Start with an initial partition Π with two group: F and S-F (aceepting and nonaccepting)
- Step 2
  - Split Procedure
- Step 3
  - If ($\Pi_{new} = \Pi$)

    $\Pi_{final} = \Pi$ and continue step 4

    else

    $\Pi = \Pi_{new}$ and go to step 2
- Step 4
  - Construct the minimum-state DFA by Π_{final} group.
  - Delete the dead state

Split Procedure

```
initially, let Π_new = Π;
for ( each group G of Π ) {
        partition G into subgroups such that two states s and t
                are in the same subgroup if and only if for all
                input symbols a, states s and t have transitions on a
                to states in the same group of Π;
        /* at worst, a state will be in a subgroup by itself */
        replace G in Π_new by the set of all subgroups formed;
}
```

Minimizing the DFA

- DFA D=({0,1,2,3,4,5}, {a,b}, δ, 0, {0,1}),其中δ见表

states	a	b
0	1	2
1	1	4
2	1	3
3	3	2
4	0	5
5	5	4

Step 1:
A={0,1}, B={2,3,4,5}。

States	partition	a	b
0	A	1(A)	2(B)
1	A	1(A)	4(B)
2	B	1(A)	3(B)
3	B	3(B)	2(B)
4	B	0(A)	5(B)
5	B	5(B)	4(B)

Major operation: partition states into equivalent classes according to: final / non-final states; transition functions

Minimizing the DFA

- DFA D=({0,1,2,3,4,5}, {a,b}, δ, 0, {0,1}),

states	partition	a	b
0	A	1(A)	2(B)
1	A	1(A)	4(B)
2	B	1(A)	3(B)
3	B	3(B)	2(B)
4	B	0(A)	5(B)
5	B	5(B)	4(B)

states	a	b
0	1	2
1	1	4
2	1	3
3	3	2
4	0	5
5	5	4

Cannot be divided any more

states	partition	a	b
0	A	1(A)	2(B)
1	A	1(A)	4(B)
2	B	1(A)	3(C)
3	C	3(C)	2(B)
4	B	0(A)	5(C)
5	C	5(C)	4(B)

Minimizing the DFA

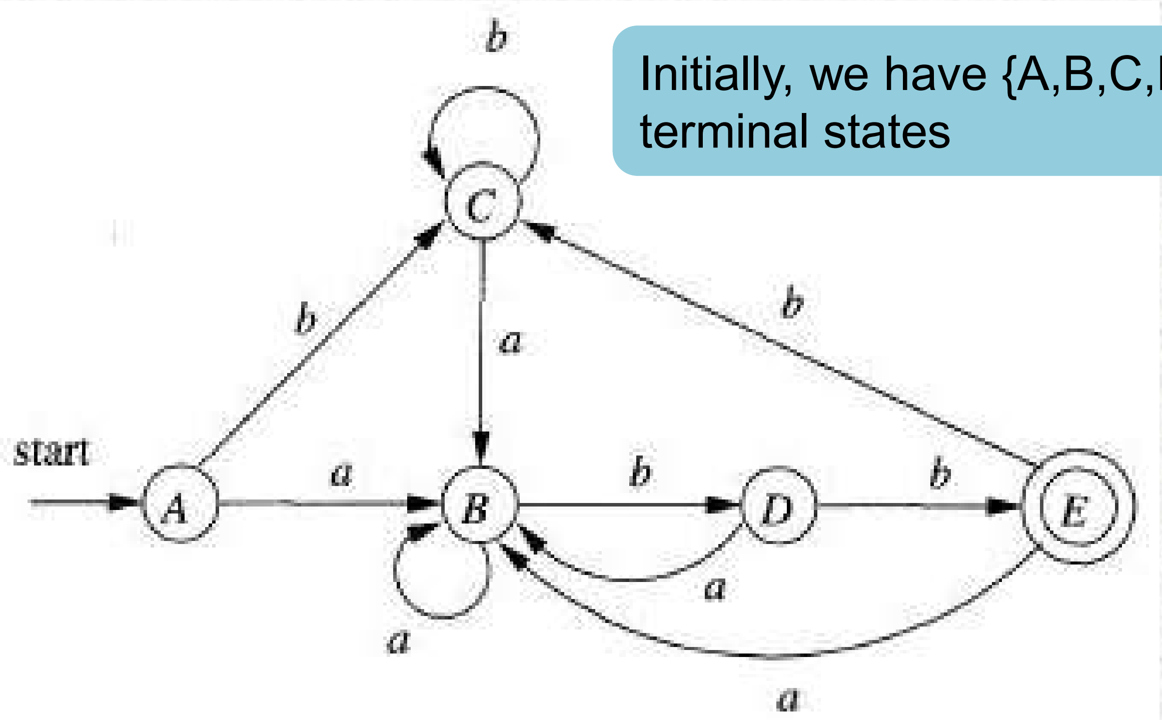
- **DFA D=({0,1,2,3,4,5}, {a,b}, δ, 0, {0,1}) is minimized to:**

DFA D′=({A,B,C}, {a,b}, δ， A， {A})， where δ is defined as follows

state	a	b
A	A	B
B	A	C
C	C	B

Minimizing the DFA-Example

- r=(a|b)*abb



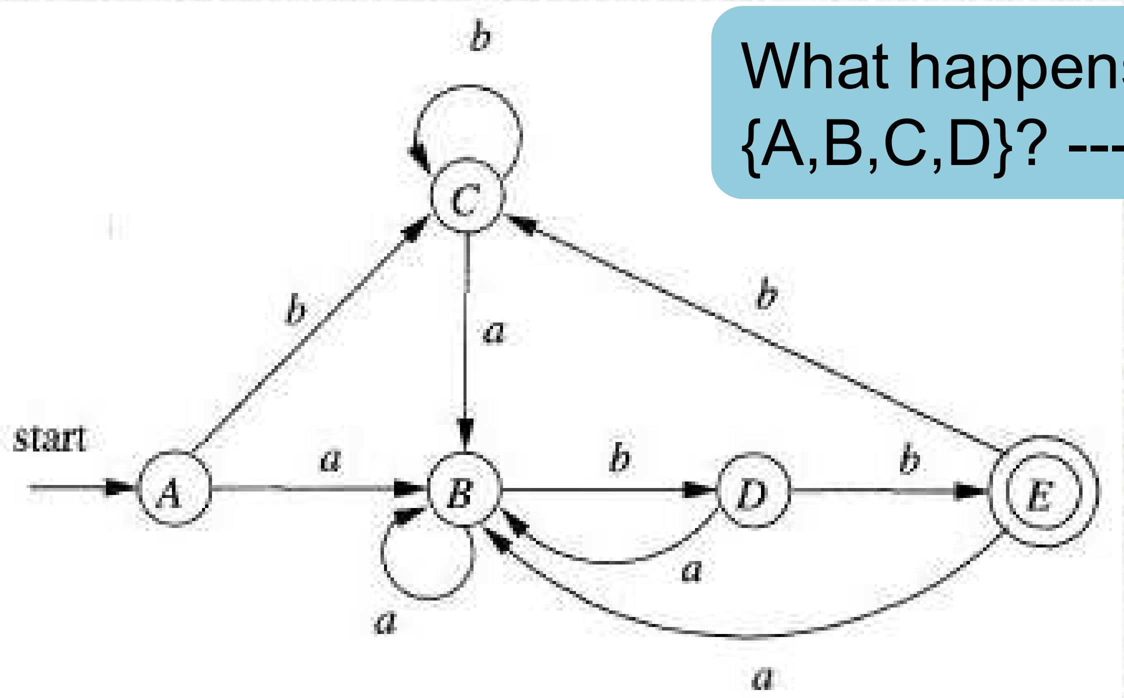
Initially, we have {A,B,C,D},{E}, which are for non-terminal and terminal states

{E} is not dividable, so we only consider {A, B, C, D}

Minimizing the DFA-Example

Is {A,B,C,D} dividable?

- **r=(a|b)*abb**

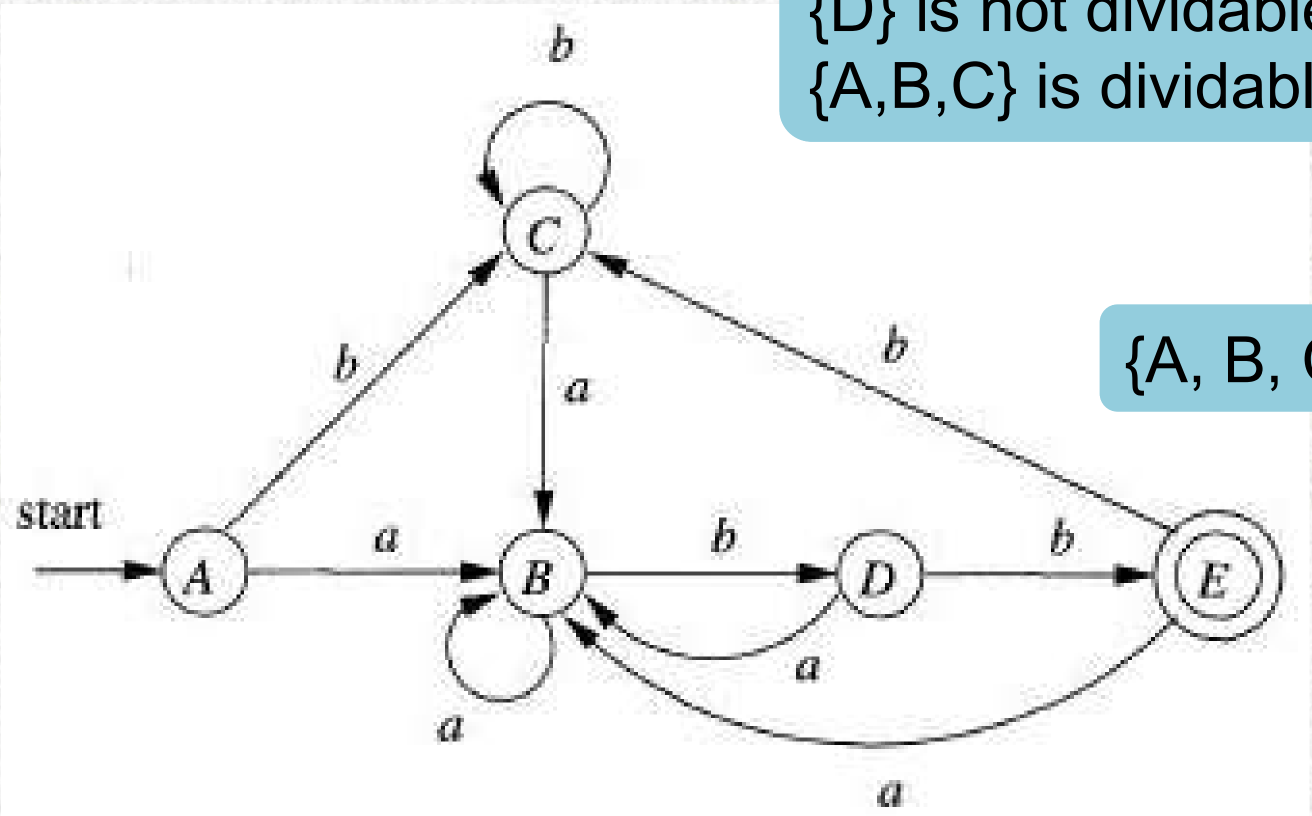


What happens when take in a under {A,B,C,D}? --- still with {A, B, C, D}

What happens when take in b under {A,B,C,D}? --- becomes {A,B,C}，{D}

Minimizing the DFA-Example

- r=(a|b)*abb



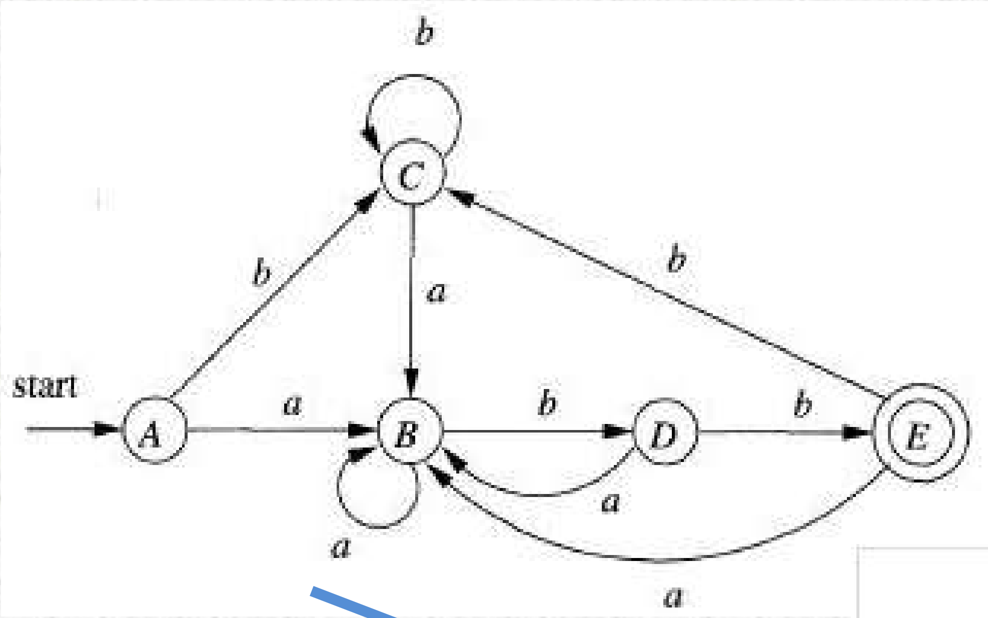
{D} is not dividable, so let us see whether {A,B,C} is dividable?

{A, B, C} becomes {A,C},{B}

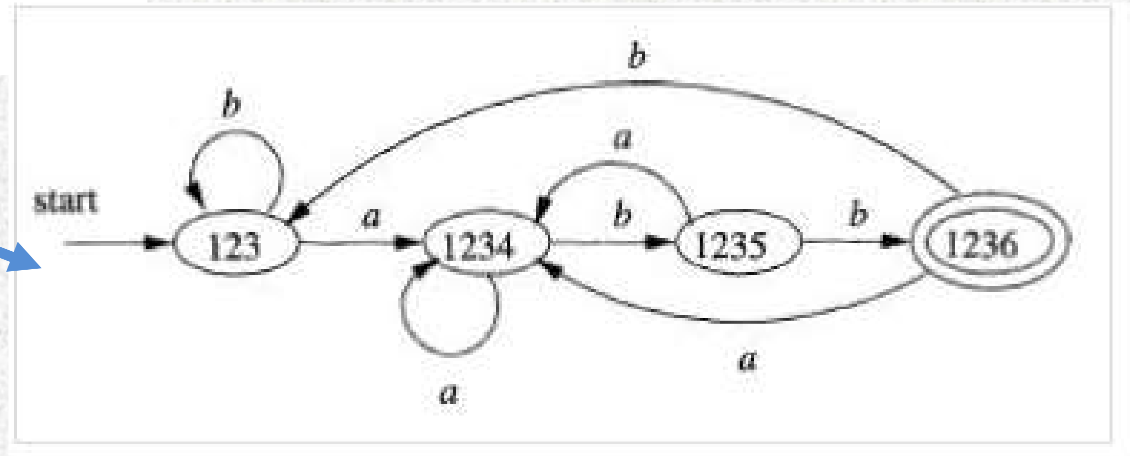
Minimizing the DFA-Example

- r=(a|b)*abb

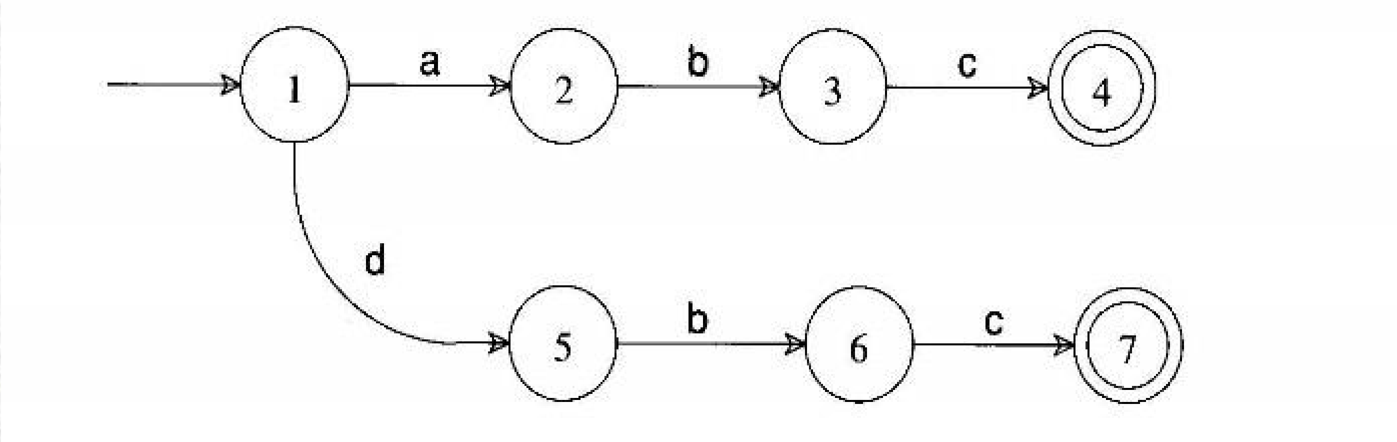
{A,C} is dividable?



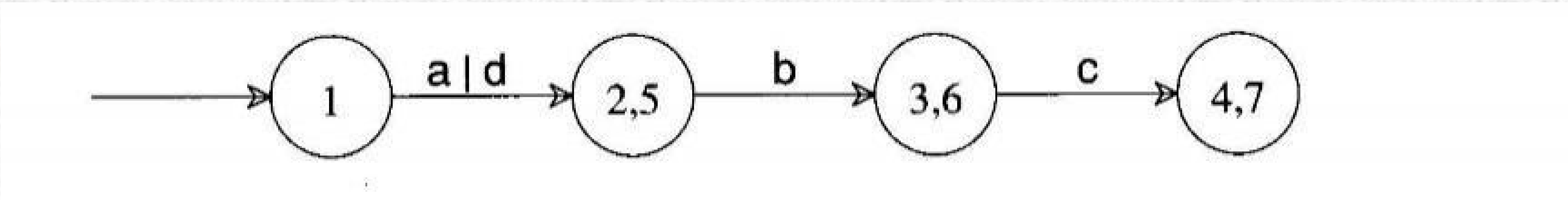
Finally, we have {A,C},{B},{D},{E}



Example

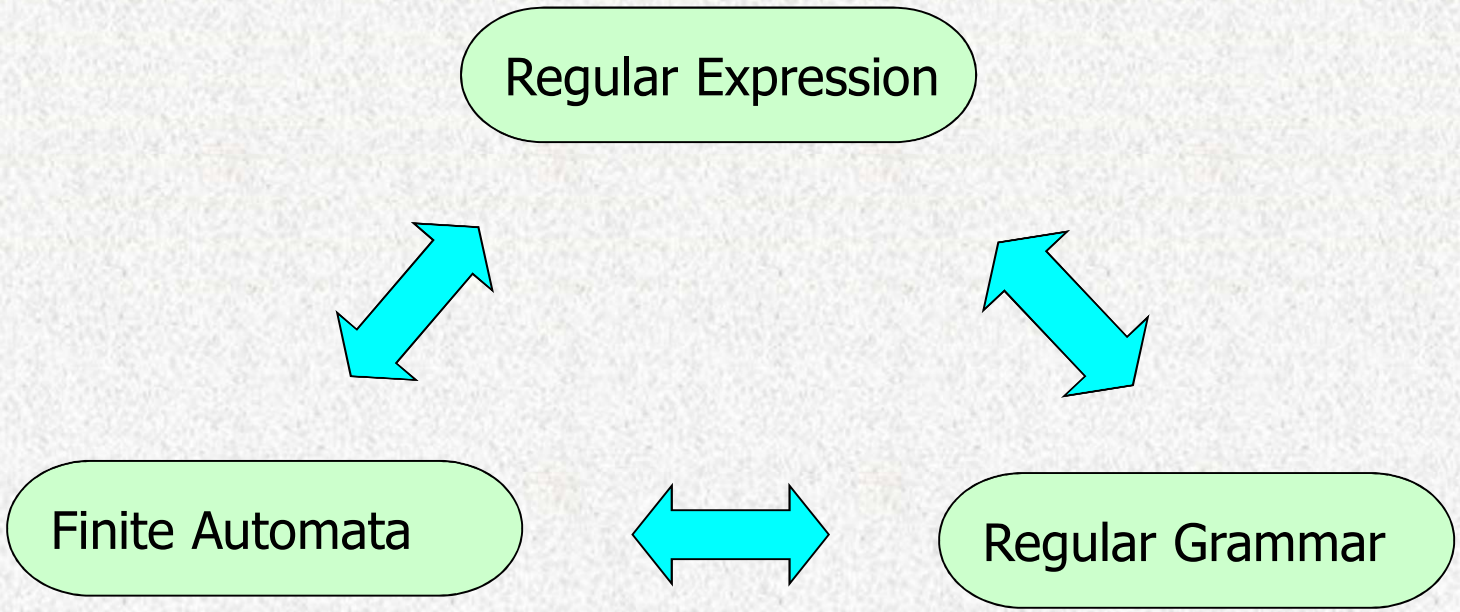


- initially, two sets {1, 2, 3, 5, 6}, {4, 7}.
- {1, 2, 3, 5, 6} splits {1, 2, 5}, {3, 6} on c.
- {1, 2, 5} splits {1}, {2, 5} on b.



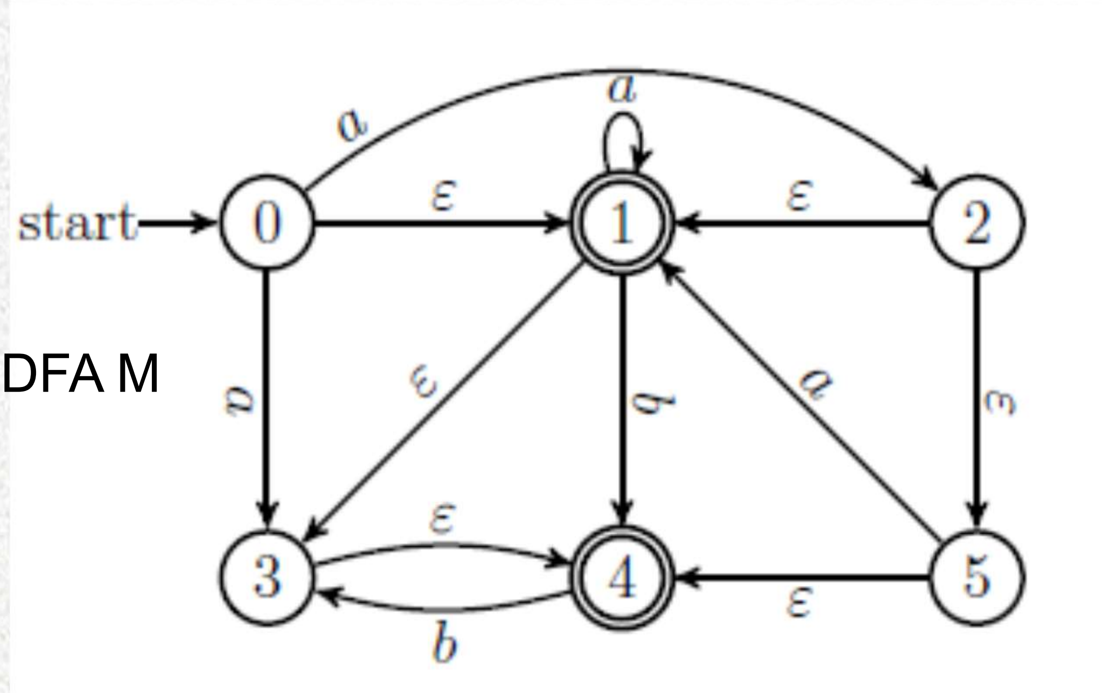
RE v.s. NFA/DFA

- RE， DFA(NFA)， L(RE) are equivalent to each other



Exercise

- Given an NFA N



(1) Simulate the NFA on input “aaabb”

(2) Convert the NFA N to its equivalent DFA M

(3) Minimize the DFA M

(4) Describe what can this DFA/NFA accept in natural language

(5) Write down the regular expression re, such that L(re) = L(N)

Homework-W3

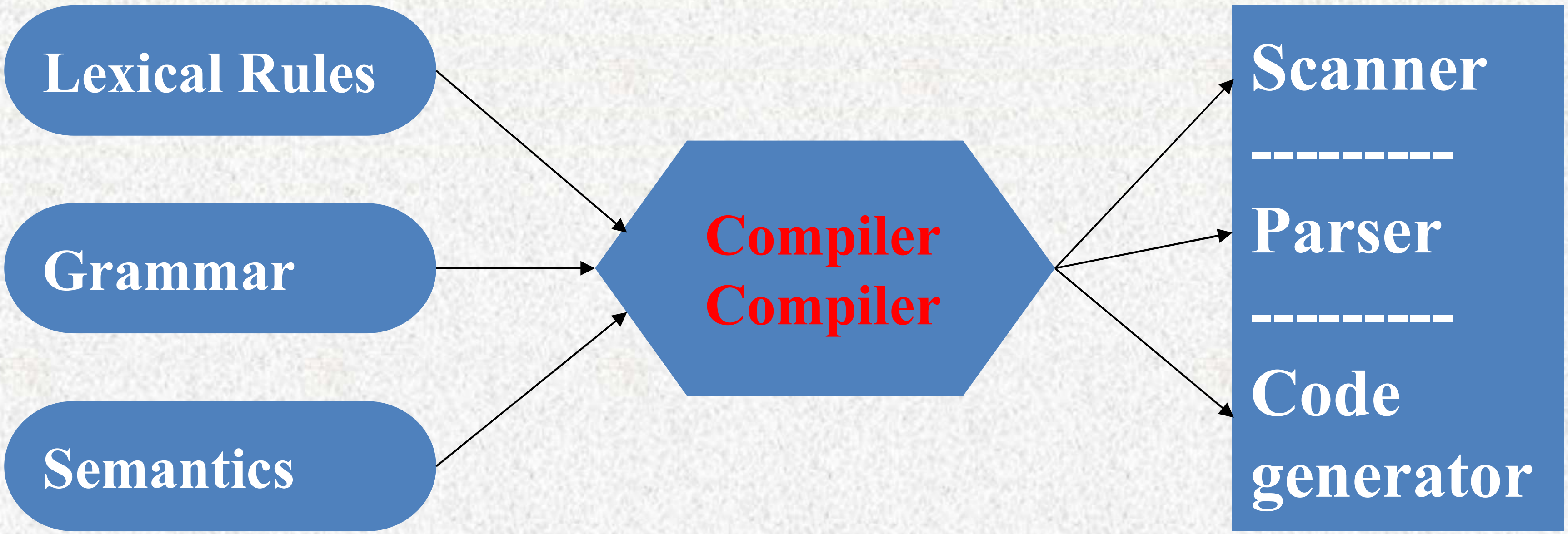
Homework – week 3

- pp. 125, Exercise 3.3.5 (c)(d)(f)(h)
- pp.152, Exercise 3.6.5
- pp. 166, Exercise 3.7.1 (b), Exercise 3.7.2 (b), Exercise 3.7.3 (d)
- pp. 172, Exercise 3.8.1
- pp.187, Exercise 3.9.4

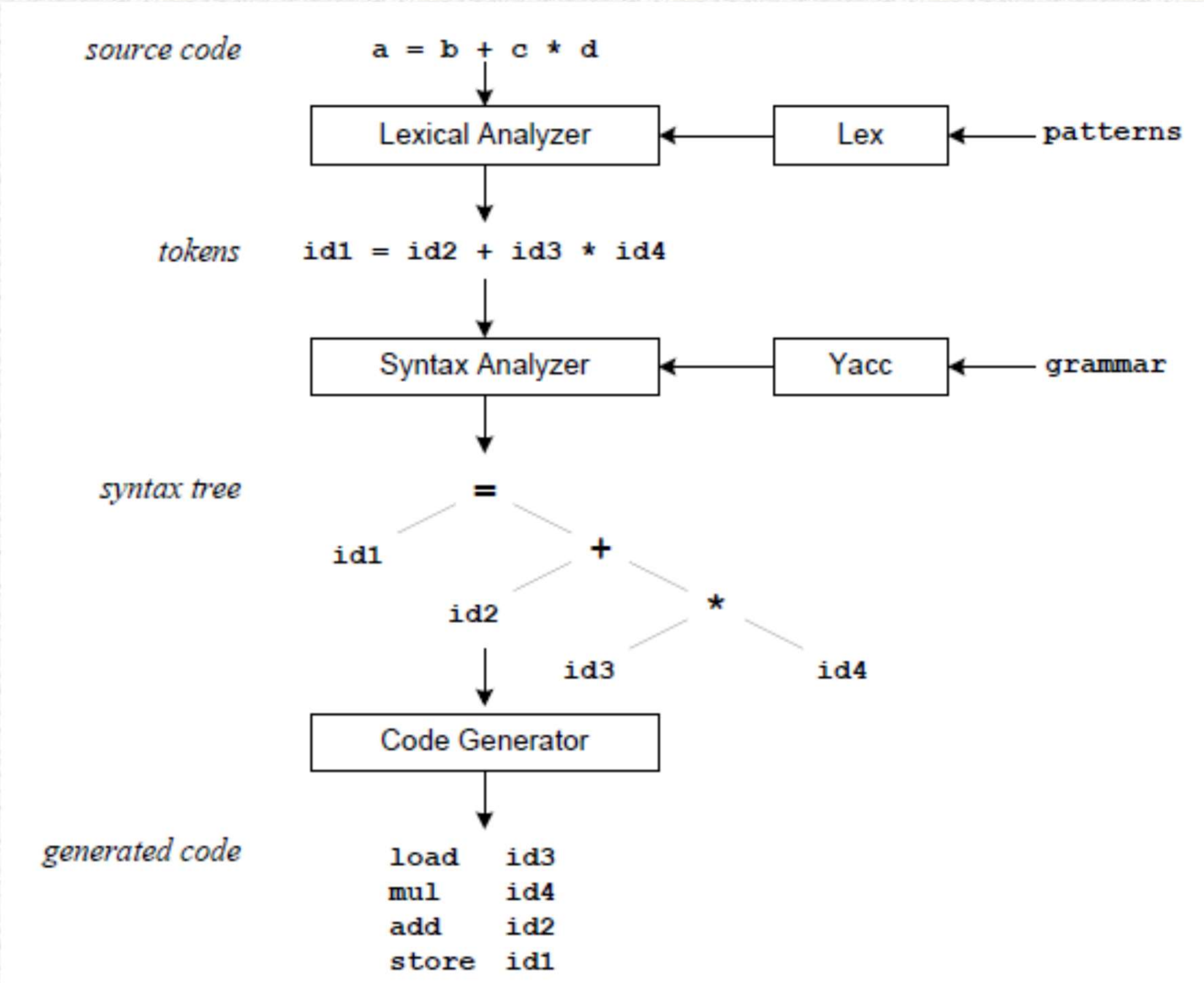
Lexical Analyzer Implementation

Overview

- Writing a compiler is difficult requiring lots of time and effort
- Construction of the scanner and parser is routine enough that the process may be automated



Overview


source code
a = b + c * d
Lexical Analyzer
Lex
patterns
tokens
id1 = id2 + id3 * id4
Syntax Analyzer
Yacc
grammar
syntax tree
=
id1
+
id2
*
id3
id4
Code Generator
generated code
load id3
mul id4
add id2
store id1

LEX

- Lex is a scanner generator
  - Input is description of patterns and actions
  - Output is a C program which contains a function yylex() which, when called, matches patterns and performs actions per input
  - Typically, the generated scanner performs lexical analysis and produces tokens for the (YACC-generated) parser

YACC

- What is **YACC** ?
  - **Tool which will produce a parser for a given grammar.**
  - YACC (Yet Another Compiler Compiler) is a program designed to compile a LALR(1) grammar and to produce the source code of the syntactic analyzer of the language produced by this grammar
  - Input is a grammar (rules) and actions to take upon recognizing a rule
  - Output is a C program and optionally a header file of tokens

LEX and YACC: a team

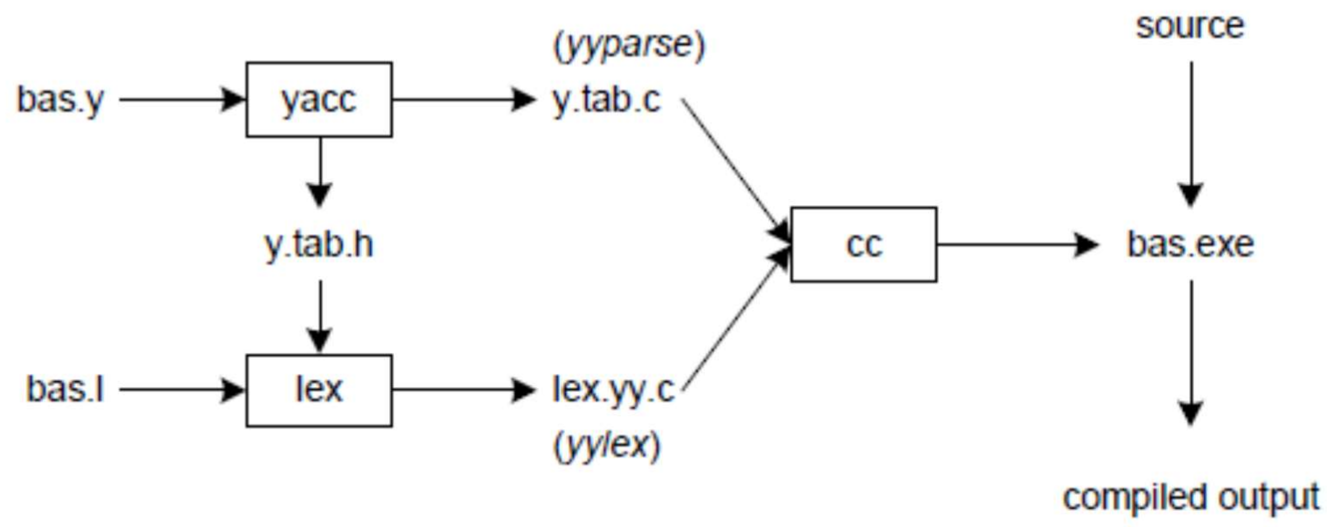


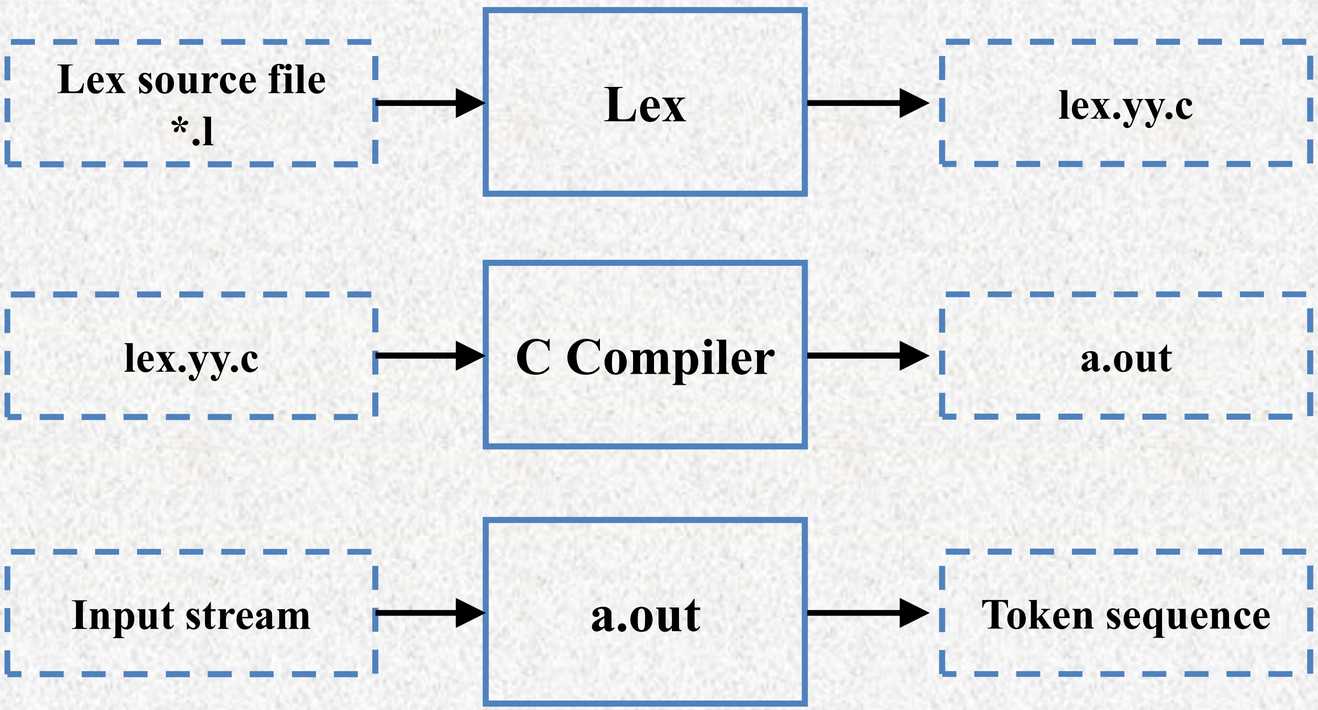
Figure 2: Building a Compiler with Lex/Yacc

```
yacc -d bas.y                   # create y.tab.h, y.tab.c
lex bas.l                       # create lex.yy.c
cc lex.yy.c y.tab.c -obas.exe   # compile/link
```

Availability

- lex, yacc on most UNIX systems
- bison: a yacc replacement from GNU
- flex: *f*ast *lex*ical analyzer
- BSD yacc
- Windows/MS-DOS versions exist

Lex



Create your lexical analyzer with Lex

Structure of Lex source file

```
Declaration
%%
Translation
   rules
%%
Auxiliary
functions
```

```
%{ constant
}%
```

Regular definition

Pattern {Action}:

- Pattern is a regular expression or regular definition
- Action is in C, describing the actions after matching the regular expression

```
%{ ID,NUM,IF,ADD
}%
```

```
letter  [A-Za-z]
digit  [0-9]
id     {letter}({letter}|{digit})*
num {digit}+
```

```
if   {return (IF);}
+   {return(ADD);}
{id} {yylval = strcpy(yytext,
       yylength); return(ID);  }
{num} {yylval = Change();
          return(NUM);}
```

Functions used in the action

```
int  Change()
{  /*Convert string into integer*/
}
```

yylval: value of the token
yytext: lexeme of the token
yyleng: length of the lexeme

Example: LEX

```
%{
#include <stdio.h>
#include "y.tab.h"
%}
id          [_a-zA-Z][_a-zA-Z0-9]*
wspc        [ \t\n]+
semi        [;]
comma       [,]
%%
int         { return INT; }
char        { return CHAR; }
float       { return FLOAT; }
{comma}     { return COMMA; }          /* Necessary? */
{semi}      { return SEMI; }
{id}        { return ID;}
{wspc}      {;}
```

Example: Definitions

```
%{
#include <stdio.h>
#include <stdlib.h>
%}
%start  line
%token  CHAR, COMMA, FLOAT, ID, INT, SEMI
%%
```

Example: Rules

```
/* This production is not part of the "official"
 * grammar. It's primary purpose is to recover from
 * parser errors, so it's probably best if you leave
 * it here. */

line :  /* lambda */
     | line decl
     | line error {
        printf("Failure :-(\n");
        yyerrok;
        yyclearin;
            }
     ;
```

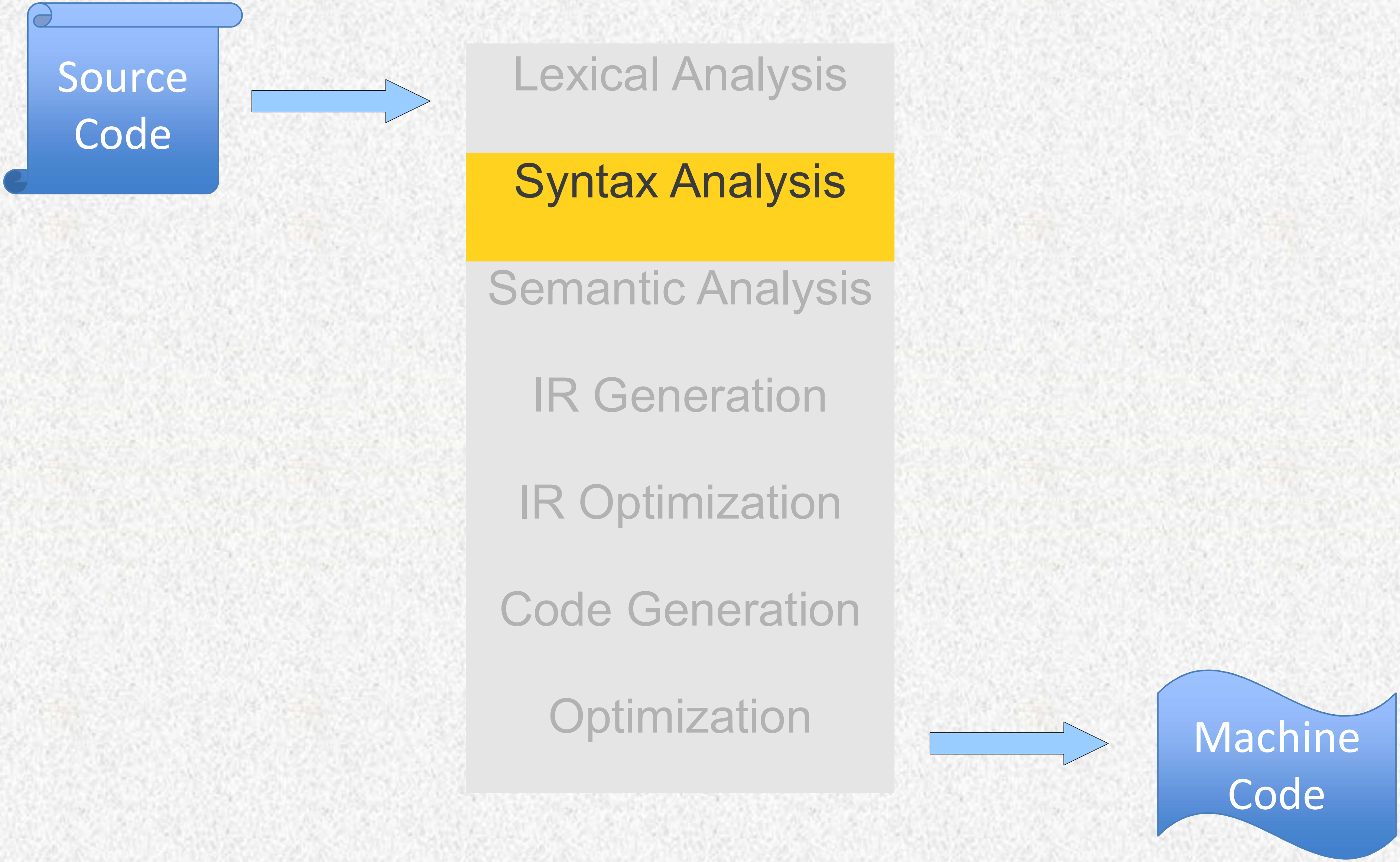
Example: Rules

```
decl :   type ID list { printf("Success!\n"); } ;

list :   COMMA ID list
     | SEMI
     ;
type :    INT | CHAR | FLOAT
     ;

%%
```

Example: Supplementary Code

```
extern FILE *yyin;
main()
{
    do {
        yyparse();
    } while(!feof(yyin));
}
yyerror(char *s)
{
   /* Don't have to do anything! */
}
```

Next Time


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code